Use of Bootstrapping in Stochastic Reserving

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Outstanding Claims Liabilities

- as at the valuation date, outstanding claims liabilities (OSCLs) refer to all claims incurred prior to the valuation date but not paid by the valuation date
- actuaries need to assess both the central estimate (i.e. expected value) and variability of the OSCL for each line of business

Deterministic Methods

- previously, actuaries only need to use deterministic methods to assess OSCLs
- chain ladder, payments per claim incurred (PPCI), payments per claim finalised (PPCF), Bornhuetter-Ferguson
- produce a single point estimate
- simple to use in spreadsheets and providing sensible results
- apply a few methods at the same time, sensitivity analysis, scenario analysis

Regulatory Changes

- Australian Prudential Standard GPS 310 stipulates that OSCLs must be valued at 75^{th} percentile
- risk margin = 75^{th} percentile – mean
- risk margin is subject to a minimum of one half of standard deviation
- risk margin is usually expressed as a percentage of central estimate

Regulatory Changes

- Singapore Actuarial Society (SAS) Guidance Note GN G01 requires setting up provisions for adverse development (PAD), driven by Section 37 (1)(b) of Insurance Act
- PAD = 75^{th} percentile – mean

Risk Margin

- allow for uncertainty inherent in OSCL assessment
- process error : future claim payments are random and unknown
- parameter error : uncertainty in parameter estimation
- model error : reserving methods adopted may not reasonably reflect underlying claims development mechanism

Risk Margin

- data error : mistakes or omissions in data
- future trends variability : parameters, inflation, interest rate, exchange rate, claims run-off patterns, claims management, reinsurance arrangements, policy conditions, exposure, business mix, staff departure, legislation, insurance market cycle, and technology may change over time
- reinsurance risk : reinsurer default

Stochastic Reserving Methods

- actuaries now have to use stochastic reserving methods as well
- bootstrapping, stochastic chain ladder, Mack model, Bayesian Markov chain Monte Carlo (MCMC) simulation
- produce both the central estimate and variability
- require more computing resources
- hitherto, literature focuses only on process error and parameter error

Bootstrapping

- relatively easy to use and understand
- can readily be programmed into common computer languages, e.g. Excel VBA
- can be combined with some deterministic methods, e.g. chain ladder
- include process error and parameter error
- we suggest below how to include model error as well

Claims Run-Off Triangle

- assume all claims are settled in n years
- there are n years of past claims data
- $X_{i,j}$ is incremental claim amount of accident year i and development year j
- $C_{i,j}$ is cumulative claim amount of accident year i and development year j
- R_i is OSCL of accident year i
- R is total OSCL

Claims Run-Off Triangle

- e.g. $n = 10$

	1	2	3	4	5	6	7	8	9	10
1	$X_{1,1}$	$X_{1,2}$	$X_{1,3}$	$X_{1,4}$	$X_{1,5}$	$X_{1,6}$	$X_{1,7}$	$X_{1,8}$	$X_{1,9}$	$X_{1,10}$
2	$X_{2,1}$	$X_{2,2}$	$X_{2,3}$	$X_{2,4}$	$X_{2,5}$	$X_{2,6}$	$X_{2,7}$	$X_{2,8}$	$X_{2,9}$	
3	$X_{3,1}$	$X_{3,2}$	$X_{3,3}$	$X_{3,4}$	$X_{3,5}$	$X_{3,6}$	$X_{3,7}$	$X_{3,8}$		
4	$X_{4,1}$	$X_{4,2}$	$X_{4,3}$	$X_{4,4}$	$X_{4,5}$	$X_{4,6}$	$X_{4,7}$			
5	$X_{5,1}$	$X_{5,2}$	$X_{5,3}$	$X_{5,4}$	$X_{5,5}$	$X_{5,6}$				
6	$X_{6,1}$	$X_{6,2}$	$X_{6,3}$	$X_{6,4}$	$X_{6,5}$					
7	$X_{7,1}$	$X_{7,2}$	$X_{7,3}$	$X_{7,4}$						
8	$X_{8,1}$	$X_{8,2}$	$X_{8,3}$							
9	$X_{9,1}$	$X_{9,2}$								
10	$X_{10,1}$									

Claims Run-Off Triangle

- upper left triangle ($i + j \leq n + 1 = 11$) is past claims data
- missing lower right triangle ($i + j > n + 1 = 11$) is unknown OSCL
- 1 diagonal of claims data is added to the triangle each calendar year

Claims Run-Off Triangle

- $C_{i,j} = X_{i,1} + X_{i,2} + \ldots + X_{i,j}$
- $R_i = X_{i,n+2-i} + X_{i,n+3-i} + \ldots + X_{i,n}$
- $R = R_2 + R_3 + \ldots + R_n$

Chain Ladder Projections

- development factors :

$$f_j = \sum_{r=1}^{n-j} C_{r,\,j+1} \Big/ \sum_{r=1}^{n-j} C_{r,\,j}$$

- projections start from the most recent diagonal $(i + j = n + 1)$
- projected values of future claims :

$C_{i,j} = C_{i,j\text{-}1} \text{ x } f_{j\text{-}1}$ $(i + j > n + 1)$

- fitted values of past claims :

$C_{i,j} = C_{i,j+1} / f_j$ $(i + j < n + 1)$

Residuals from Chain Ladder

- fitted values or projected values :

  $X_{i,j} = C_{i,j} - C_{i,j-1}$
- for each cell (i , j) of past claims:

  residual = (observed – fitted) / zigma
- for each development year j :

  $zigma^2 = \sum_{dy}(\text{observed} - \text{fitted})^2 / (n_{dy} - 1)$
- assume residuals are iid

Bootstrapping

- first step allows for parameter error
- resample residuals with replacement
- pseudo past claims ($i + j \leq n + 1$) :

  resampled residual x zigma + fitted value
- apply chain ladder to these pseudo past claims to obtain bootstrap forecasts

Bootstrapping

- second step allows for process error
- resample residuals with replacement again
- pseudo future claims ($i + j > n + 1$) :

  resampled residual x zigma + projected value

Bootstrapping

- repeat say 1,000 times
- draw inferences from the simulated samples of the difference between bootstrap forecast values and pseudo future claims
- determine risk margin based on these inferences

Example – Claim Payments

i \ *j*	1	2	3	4	5	6	7	8	9	10
1	357,848	766,940	610,542	482,940	527,326	574,398	146,342	139,950	227,229	67,948
2	352,118	884,021	933,894	1,183,289	445,745	320,996	527,804	266,172	425,046	
3	290,507	1,001,799	926,219	1,016,654	750,816	146,923	495,992	280,405		
4	310,608	1,108,250	776,189	1,562,400	272,482	352,053	206,286			
5	443,160	693,190	991,983	769,488	504,851	470,639				
6	396,132	937,085	847,498	805,037	705,960					
7	440,832	847,631	1,131,398	1,063,269						
8	359,480	1,061,648	1,443,370							
9	376,686	986,608								
10	344,014									

Results

- total OSCL :

  central estimate = 18,680,856

  risk margin = 1,502,161 (8.0%)

Bootstrapping – Model Error

- we suggest using bootstrapping to consider potential model misspecification, in addition to process error and parameter error
- find an initial set of competing models that fit the data well
- adopt a set of statistical and judgmental criteria Π to select between the competing models in the bootstrapping process

Bootstrapping – Model Error

- in the bootstrapping process, fit the competing models in turn to the pseudo past claims generated
- use the best performing model (based on Π) to produce bootstrap forecasts
- this modification allows for both parameter error and model error

Example – No of Claims Reported

i \ *j*	1	2	3	4	5	6	7	8	9	10
1	195	297	181	55	22	16	16	8	8	5
2	192	289	178	53	23	16	15	9	8	
3	178	268	165	50	20	14	15	7		
4	172	261	162	48	21	14	15			
5	226	339	211	64	26	19				
6	266	401	247	74	30					
7	262	393	244	73						
8	244	368	230							
9	224	338								
10	189									

Results

- PPCI vs chain ladder
- single criteria $\prod = \sum|\text{residual}|$
- total OSCL :

  central estimate = 18,757,878

  risk margin = 1,997,661 (10.6%)
- increase in risk margin can be deemed as approximate extra allowance for model error resulting from selection between the prior competing models

Bootstrapping – Model Error

- framework of our approach is general in nature
- can be adapted to a variety of models, claims data, and selection criteria
- any statistical or judgmental criteria, provided it is feasible to automate their exercising, can be implemented