Stochastic Reserving Techniques

(Reserving Techniques in Use by Actuaries Today)

Matthew Maguire
FIA, FIAA, FSAS

SAS General Insurance

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Table of Contents

Introduction

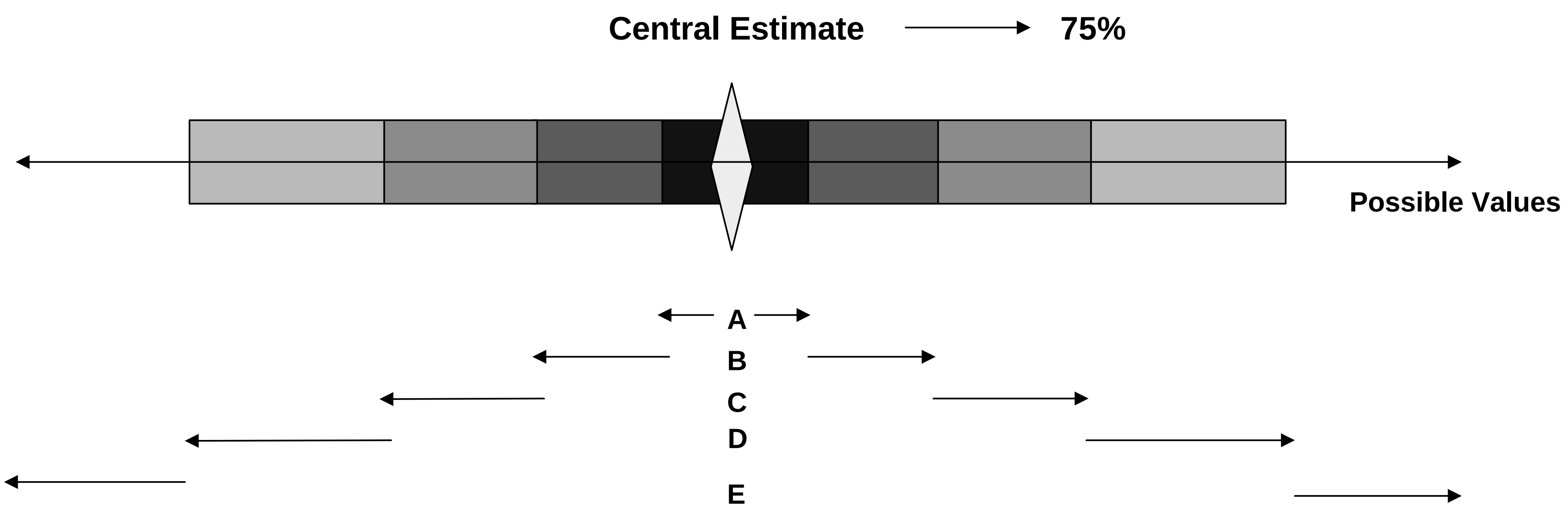
- Predicting the future “not an exact science”
- Reserves are funds set aside to meet future obligations
- These future obligations are the result of a large number of random processes
- Can only determine an estimate. All estimates should also convey the confidence or certainty of the estimate.

What is an Actuary's "Best Estimate"?

- It is the subjective derivation of the mean of all possible outcomes, taking into account all available information about the business being analysed.
- Allows for subjective interpretation and choice of methods and models.
- Generally excludes an allowance for events not reflected in the data
  - Eg unanticipated new forms of latent claims
- Uncertainty allows for different judgements to be made on how the future will unfold.
- A variety of reasonable best estimates is possible.

Ranges

Claims Liability

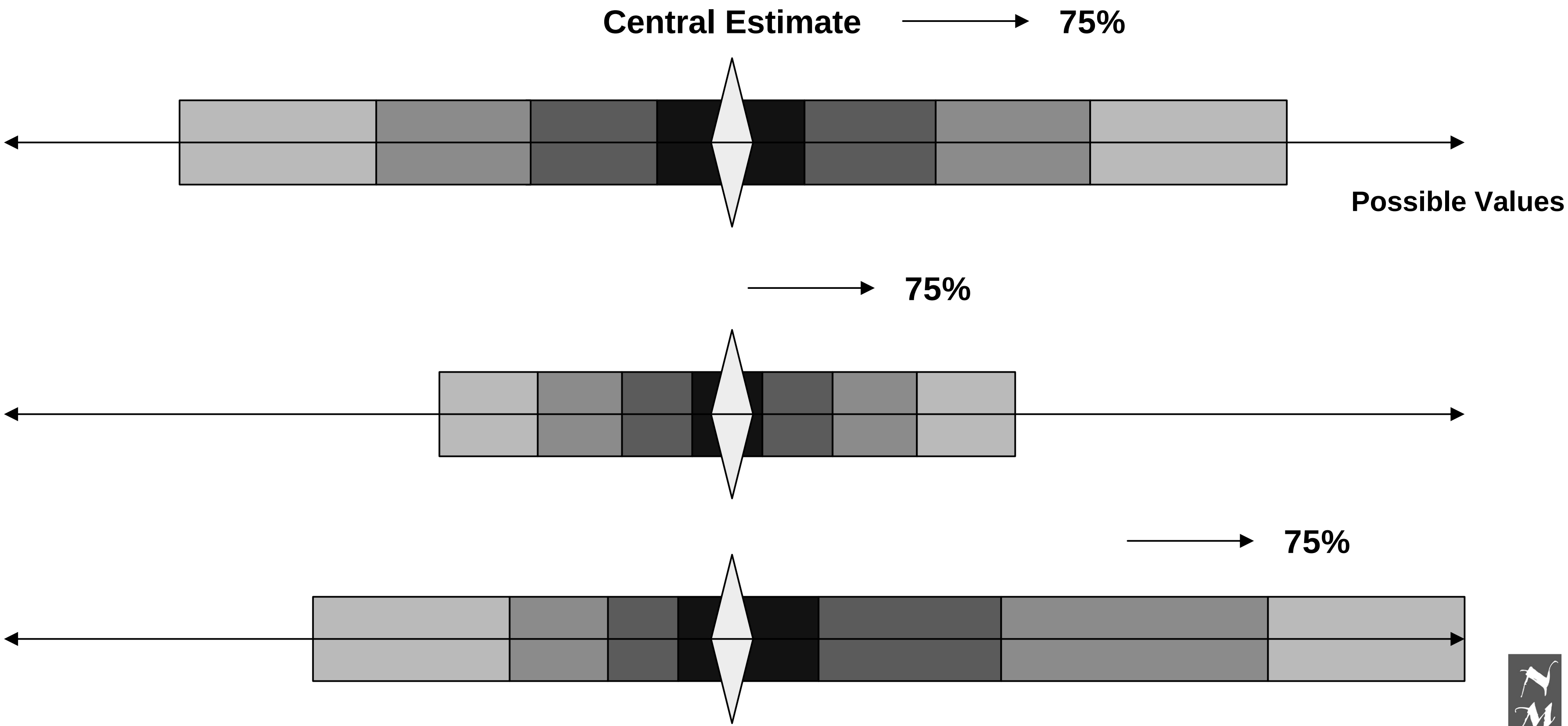


A: Comfortable B: Reasonable C: Not Unreasonable D: Surprising E: Not Possible

Subjective

Ranges

Claims Liability



Purpose of Reserving

- Develop a value for reserves

to

- Ensure sufficient funds to pay out claims

by

- Ensuring profit is not released prematurely

Purpose of Reserving

- Traditional Techniques determine a “Best” or “Point” estimate

- Actuaries have become more interested in developing methods for quantifying the uncertainty of these estimates

- RBC formalises this by using a defined “Probability of Sufficiency”

What is Stochastic Reserving

- Stochastic Reserving is exactly what it says
    - Treating the estimated reserve as a random variable
- General approach is to select a “Best” or “Central” estimate of the values …
- … then determine the variability and select some “Confidence Levels”

Sources of Uncertainty

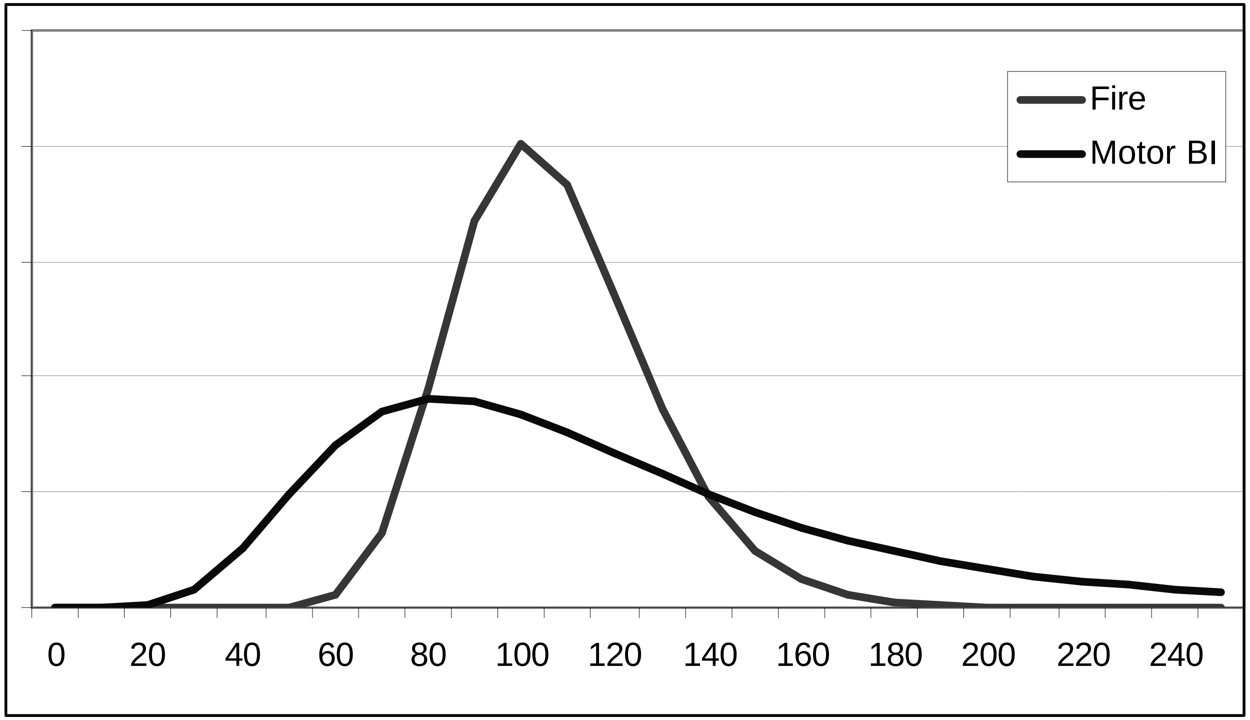
- How many claims will there be?
- How big will each claim be?
- When will the claims be paid?

Sources of Uncertainty

- Random Variance
- Changes in the environment
  - Notification Delays
  - Legal changes / Court rulings
  - Changes in Society's "Propensity to Claim"
  - Claim inflation rates
- Changes in Company Processes
  - Claims reporting, claims controls
- Projection Process Uncertainty
  - Model error, Parameter error

Understanding the Business

- Expected Claim Liability:
  - Fire: 100m
  - Motor BI: 100m



Very different views of uncertainty

A "Point Estimate" does not convey this uncertainty

Regulatory Requirements

- MAS and BNM both require reserves to be calculated at 75% sufficiency for RBC purposes
  - Capital calibrated to the assumption of given level of reserve sufficiency
  - Allows greater comparison between companies
  - Will not produce more stable profits as the 75% value just as volatile as the central estimate

Reserving Techniques

- Most methods based on assumptions on the underlying shape of the claims run-off

- These assumptions define a mathematical model of the run-off

- Stochastic methods model the variations in the patterns

Benefits of Stochastic Methods

- Can estimate the likely magnitude of Random Variation
- Can apply statistical tests to the modelling process to verify assumptions
- Develop an understanding of the variability of the claims process
- Can design a model so that results are based upon the more credible data points

Statistical Models

- Three components
  1. A Statistical Model
  2. A way of fitting the model to past data
  3. A justification that the model will predict the future
- Using models for prediction requires:
  - That the model describes behaviour in the future (irrespective of its past experience)
  - The parameters have been correctly determined.

Issues in Modelling

- Process Error
  - Future payments are Random and Unknown
- Parameter Error
  - Uncertainty in parameter estimation
- Model Error
  - Reserving method adopted do not reflect the underlying claims development mechanism

Mack Method

- Example of an Analytical Method
- Based on the Chain Ladder approach
- Calculates error terms in triangles
- Estimates Process and Parameter errors
- Assumes a lognormal distribution for percentiles
- Easy to implement in a spreadsheet

Mack Method - Assumptions

- Run-off pattern is the same for each origin period
- Future development for a cohort is independent of historical factors
  - Ie high factors in one period do not imply high or low factors in a following period
- The variance of the cumulative claims to development time ‘t’ is proportional to the cumulative claims amount to time ‘t-1’

Bootstrapping

- Refer to Jackie Li SASGI 2009
  - Use of Bootstrapping in Stochastic Reserving
- Model can be any statistical or judgemental criteria
  - Provided it is feasible to automate
  - If significant judgement involved then can not be automated for boot-strapping
- Note: if model is flawed then re-sampling will not help.

Bootstrapping - Steps

1. Start with a triangle
2. Fit a model (Chain Ladder, PPCI etc)
3. Determine Residuals
4. Sample the residuals (with replacement)
5. Recreate the triangle with pseudo data
6. Reapply the model to obtain forecast
7. Repeat steps 4 to 6 “many” times

Implied Development Factor Analysis

1. Complete analysis using any approach
2. Review history of development factors (D.F.)
   - Eg Yr 1 to Ultimate, Yr 2 to Ultimate
3. Determine Mean and SD of historical development
4. Apply C of V to current accident year D.F.
5. Apply a distribution to parameters (eg Log-Normal)
6. Simulate for each accident year
7. Sum the simulations for each Accident Year

Frequency / Severity Forecasting

1. Complete analysis using prefferred approach

2. Complete analysis of ultimate number of claims

3. Determine implied average size of claims to be closed

4. Assume a distribution for the frequency and severities

5. Simulate and combine

Probabilistic Trend Family

- Examines trends in Development Year and Calendar Year
- Fits lognormal distributions to each cell and projects lognormal distributions to the future
- Uses regression on the logs of residuals
- Process is to retain only the *significant* parameters
- Percentiles can be derived from combining the individual distributions

Variability vs Uncertainty

- Not interchangeable terms
- Variability
  - Effect of Chance
  - A function of the process
  - Not reduceable through further study or measurement
- Uncertainty
  - Lack of knowledge about parameters or model structure
  - May be reduced through further study

Variability vs Uncertainty – Example 2

- A symmetric coin is tossed 100 times
- The mean number of heads, the SD is 5
  - This is known
  - There is no uncertainty about the coins variability
- A 100% CI for the mean is 50
- A 95% prediction interval for the outcome is 40 – 60
- This 95% prediction interval cannot be shortened

Variability vs Uncertainty – Example 2

- A real coin has an unknown probability of a head
- The coin is tossed 10 times giving 5 heads
  - The estimate of the probability of a head is 0.5
  - But uncertain – 95% CI is [0.26 – 0.81]
- A 100% CI for the mean is 50
- 95% prediction interval for number of heads in 100 tosses is 24 - 83
- Variability range is ± 10, Uncertainty adds 30 to the range

Variability vs Uncertainty – Example 3

- A coin is tossed 10 times giving 3 heads
  - If don't know coin is fair assume a 95% CI of [0.12 – 0.65]
  - 95% PI of 100 tosses is 11 – 67
  - Central estimate = 30, 75th percentile = 49.
- Favourable history gives a 75% sufficiency below the mean
  - Highlights the importance of parameter error
  - Without uncertainty adjustment 75 percentile is 33

Model Appropriateness

- Important to test
- Plots of Residuals
- Numbers of Parameters
- Back testing
    - Fit model to old data and test reasonableness

Aggregation across Business Lines

- RBC requires 75% confidence at company level
- Summing 75% value for each class assumes 100% correlated – not likely
- But likely to be some correlation
  - Requires judgement on correlations
- Can apply stochastic technique to aggregated triangle and compare with 100% correlated value to estimate the diversification discount

Issues with Stochastic Reserving

- If triangle has a negative development factor then techniques using lognormal do not work
  - Less an issue with Paid than Reported data
- Techniques are based on data available
  - Can’t adjust for unknown claims eg latent claims
- Small data sets mean small changes in numbers can have a significant impact on distributions
- Extremes of distributions
  - 99.5% Confidence operating well beyond the limits of a standard data set

Conclusions from GIRO

- Effectiveness of Reserving Methods Working Party

1. There is no perfect method

2. Statistical Diagnosis of historical data patterns must be combined with understanding of the business for sound judgements about the future

3. Challenge is to move from historical diagnosis to future estimation via business understanding

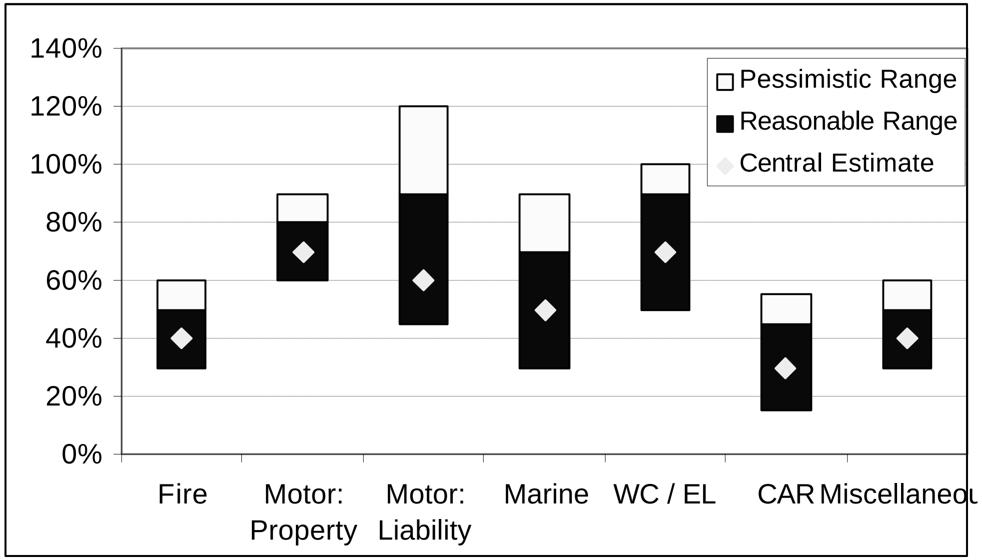
4. A good method can only take you so far

Importance of Communication

- Mathematical derivation of results can be complicated:

$$(s.e.(R))^2 = \sum_{i=2}^{I} \left\{ (s.e.(R_i))^2 + C_{iI} \left(\sum_{j=i+1}^{I} C_{jI} \right) \sum_{k=I+1-i}^{I-1} \frac{2a_k^2 / f_k^2}{\sum_{n=1}^{I-k} C_{nk}} \right\}$$

- However concepts can be explained with charts and tables



Conclusion

- In relation to Stochastic Reserving:

A good technique
does not make
a bad model good

- Given the inherent uncertainty does applying a label like “75% sufficiency” imply greater accuracy than is really possible?

Questions?

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