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# Achieving Market Consistent Pricing of Options & Bonds Where No Market Exists

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# Agenda

- + What does market consistency and fair value promise?
- + What do regulators and accountants have to say about missing market prices?
- + A review of the possible solutions
- + Valuation of fixed liabilities and yield curve extension
- + Valuation of options and 'mark-to-model' of long-dated, untraded options
- + Towards greater consistency.

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# What does market consistency and fair value promise?

# Market consistency: An emerging global standard

- + ‘Early adopters’ - Individual regulatory initiatives in the UK, Switzerland (SST), South Africa (PGN110)...
- + Solvency II Directive, Article 74: “...*calculation of technical provisions shall make use of and be consistent with information provided by the financial markets...*”
- + CFO Forum Market Consistent EV Principles:
  - Principle 12: *Economic assumptions must be internally consistent and should be determined such that projected cash flows are valued in line with the prices of similar cash flows that are traded on the capital market...*
  - Principle 13: *VIF should be discounted using discount rates consistent with those that would be used to value such cash flows in the capital markets.*

# Market consistency:

- + Alignment of accounting and regulatory valuation with economic values.
- + A foundation for closer alignment of regulatory solvency capital with economic capital.
- + Encourages economic pricing of financial products (more specifically long-term guarantees).
- + Promotes a better understanding of risk and value.
- + Promotes better quality risk management actions by firms (and appropriate intervention by regulators).

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# What do regulators & accountants have to say about missing market prices?

# IASB: Fair value measurement

*IASB Expert Advisory Panel: “Measuring and disclosing the fair value of financial instruments in markets that are no longer active”, October 2008*

- + “..the price at which an orderly transaction would take place between market participants .. [measured] ..by considering all relevant market information ..”
  - “.. market information available about identical or similar instruments. Such information to be considered includes .. prices from recent transactions in the same or a similar instrument, quotes from brokers and/or pricing services, indices and other inputs to model-based valuation techniques.”
- + “When the market for a financial instrument is no longer active.. ‘mark-to-model’ ”
  - “.. maximises the use of observable inputs and minimises the use of unobservable inputs .. Regardless of the valuation technique used, an entity takes into account current market conditions and includes appropriate risk adjustments that market participants would make, such as for credit and liquidity.”
- + “..calibrate.. the valuation model to observable market information to ensure that the model reflects current market conditions and to identify any potential deficiencies in the model.”

# Active markets

- + Active market (IASB Expert Panel):

*“A financial instrument is regarded as quoted in an active market if quoted prices are readily and regularly available from an exchange, dealer, broker, industry group, pricing service or regulatory agency, and those prices represent actual and regularly occurring market transactions on an arm's length basis.”*

- + No active market: valuation technique

*“The objective of using a valuation technique is to establish what the transaction price would have been on the measurement date in an arm's length exchange motivated by normal business considerations. Fair value is estimated on the basis of the results of a valuation technique that makes maximum use of market inputs, and relies as little as possible on entity-specific inputs.”*



# Fair value hierarchy

FAS 157 (and IFRS 7) contains an explicit three-level ‘fair value hierarchy’ which groups fair value measurements based on their observability at three levels:

Level 1: Mark-to-market using quoted prices (unadjusted) in *active markets* for identical assets or liabilities.

Level 2: Mark-to-Model using inputs other than quoted prices included within Level 1 that are observable for the asset or liability, either directly or indirectly.

Level 3: Mark-to-Model using significant unobservable inputs for the asset or liability.

QIS 4 / TS.I.A.2 ...the following hierarchy of high level principles is proposed for the valuation of assets and liabilities:

- (i) Wherever possible, a firm must use “mark to market” methods in order to measure the economic value of assets and liabilities;
- (ii) Where this is not possible, mark to model procedures should be used (marking to model is any valuation which has to be benchmarked, extrapolated or otherwise calculated from a market input). When marking to model, undertakings will use as much as possible observable and market consistent inputs;

# CEIOPS: 2009 consultation & advice

- + Solvency II: both assets and liabilities should be valued on a consistent basis using market values where they are available.
- + The value of liabilities (Technical Provisions) shall be calculated as the sum of a Best Estimate (BE) and a Risk Margin (RM). The TP should correspond to the value a third party would require to take over and meet the liabilities.
- + The BE is calculated as the expected present value of future cashflows.
  - In principle the BE should only use values from deep, liquid and transparent markets but if this is not possible other market prices may be used.
  - A market consistent asset model is a recognised technique for calculating the BE.

# CEIOPS: 2009 consultation & advice

- + The RM is a ‘cost-of-capital’ (CoC) based calculation for valuing non-hedgeable risk
  - When non-hedgeable risk is present the BE does not represent the value a third party would require to take over and meet the liabilities. The RM is the “extra bit”
  - Until recently this was thought to be for non-market risk only
  - Now extended to include non-hedgeable market risk, e.g.
    - + 60 - yr Yen or USD cashflows, 15 - yr emerging market cashflows
- + When perfect replication of liability cashflows is possible using financial instruments for which a “reliable” market value is observable then the TP shall be the market value of the replicating instruments
  - Perfect replication is very unlikely (financial instruments that reflect mortality rates?)
  - Note that “reliable” is interpreted as deep liquid and transparent (see next slide)

# Deep, liquid and transparent

Note also the potential impact of CEIOPS' CP41 & 35/09:

4.11 ... the expression 'financial instruments for which a reliable market value is observable' should be understood as financial instruments quoted in 'deep, liquid and transparent markets', which requires to meet all the following requirements:

- Market participants can rapidly execute large-volume transactions with little impact on the prices of the financial instruments used in the replications;
- Current trade and quote information of those prices is readily available to the public;
- The properties specified above are expected to be permanent.

4.12. One of the main lessons [of the crisis] is the lack of reliability of the valuations of OTC financial instruments, and the lack of transparency when financial investments are not actively traded in deep, liquid and transparent markets. In fact, a main conclusion commonly repeated in the various reports dealing with the crisis, is the necessity of limiting the scope of mark-to-model practices and non-actively traded assets.

# Deep, liquid and transparent

**CEIOPS' Advice for  
Level 2 Implementing Measures on Solvency II:**

**Technical Provisions – Article 86(c)  
Circumstances in which technical provisions shall  
be calculated as a whole**

## 4.1.2. Examples

(former CP 41)

Example	Have requirements in Article 76(4), second paragraph, of the Level 1 text been met?	Technical provisions shall be calculated:
An insurance undertaking investing in assets replicating his future cash-flows provided by a third party (e.g. investment bank).	<u>No</u> : (see paragraphs 4.8, 4.11 and 4.14) This case introduces counterparty and concentration risks with regard to the issuer of the replicating asset. Furthermore, in respect of cash-flows associated with insurance obligations it is necessary to consider 4.14.	BE + RM

- + The detail of mark-to-model practices and the risk margin calculation is not yet fully specified.
- + These issues have important implications for valuation, extrapolation methodologies and the viability of hedging strategies for insurance firms.

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# Market calibration issues emerging from Solvency II

# Solvency II issues

- + How formalised should mark to model practices be?
  - Like to look at yield curve extrapolation as an example
  - Similar issues exist around extrapolation of the equity volatility surface
- + Which prices should be used when marking to market?
  - CEIOPS states the BE should use market prices from deep, liquid and transparent markets when available. When they are unavailable “other market prices” should be used.
- + Should a RM be calculated for non-hedgeable market risk just when “other market prices” are unavailable or whenever markets aren’t deep liquid and transparent?
  - If a RM is required whenever markets aren’t deep, liquid and transparent the RM will need to be calculated frequently
- + What can we say about a CoC RM approach?
  - Do the financial markets give us any insights into CEIOPS CoC RM calculation?

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# Valuation of fixed liabilities & yield curve extension



# The issue: How to value cash flows beyond the longest bond/swap maturity

- + In highly developed fixed income markets
  - Maturity ~ 50 years for government bond and swap contracts
- + In less developed markets
  - Bond and swap markets less liquid
  - Actively traded instruments have shorter terms
  - In some cases, there may be no traded risk free instruments at all

# CEIOPS' thinking

## CP40 & 34/09

CEIOPS' Advice for  
Level 2 Implementing Measures on  
Solvency II:

Technical Provisions - Article 86 b -  
Risk-free interest rate term structure

- 3.41. There are many techniques available for extrapolating the interest rate curve. These techniques can be broken down into three categories: simple extrapolation techniques, macroeconomic techniques and parameterisation techniques. CEIOPS suggests developing further guidance at Level 3.
- 3.42. Simple extrapolation techniques require no deeper analysis of the fundamentals or shape of the curve. In its purest form, .. assumes that the final liquid data point is also the long term interest rate level. From the final liquid point onwards, the curve is .. a horizontal line.
- Advantages:* It is easy to apply. It is objective, robust and reliable.
- Disadvantages:* the entire curve would then become significantly more volatile than the macro-economic approach.

# CEIOPS' thinking

## Extracts from CP40

Consultation Paper No. 40

Draft CEIOPS' Advice for  
Level 2 Implementing Measures on Solvency II:

Technical Provisions - Article 85 b -  
Risk-free interest rate term structure

3.43. Macroeconomic extrapolation techniques involve identifying a long term equilibrium interest rate.. through economic analysis,

*Advantages:*

- Stable
- Insensitive to the downward pressure on the interest rate levels of large volumes of trade created by insurers aiming to hedge their interest rate risk
- Could be seen to be based on market consistency.

*Disadvantages:*

- long-term level is mis-stated through faulty analysis
- Method of interpolation between the final available data point
- A change in the macro-economic figures .. will cause an abrupt change in the value of liabilities.

# CEIOPS' thinking

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Technical Provisions - Article 85 b -  
Risk-free interest rate term structure

3.44. Parameterisation techniques emphasise smoothing and provide an objective construction of the term structure .. currently most used in market practice.

*Advantages:*

- Objective if fixed.
- Constant forward rate assumption is easy to apply to a curve.

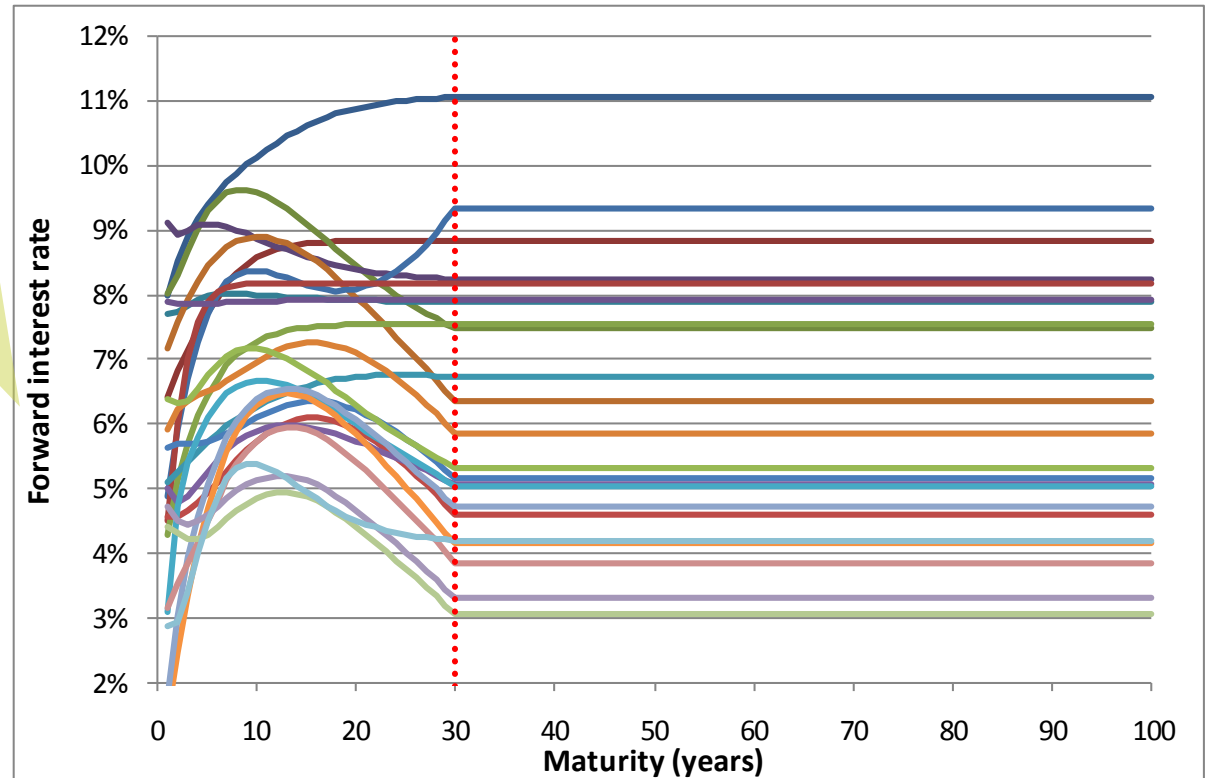
*Disadvantages:*

- Potentially sensitive to the downward pressure on the interest rate levels of large volumes of trade created by insurers
- Further disadvantages depend on the particular method chosen.
- More volatile extrapolations than the simple approximation.
- Depending on the type of Nelson-Siegel model, changes in the short end of the curve can significantly impact the long term extrapolation. Other techniques also have their own drawbacks.

# Extrapolation: *What is the problem?*

The chart plots USD government forward rates assuming constant rate beyond 30 years, End-December 1985-2007.

Although this has been suggested as a possible extrapolation method by European regulators and the CFO Forum, it is very conservative and will generate very high volatility in the MTM value of ultra long-term cash flows.



# Discount bond volatility for forward rates extrapolated from various terms at a constant level

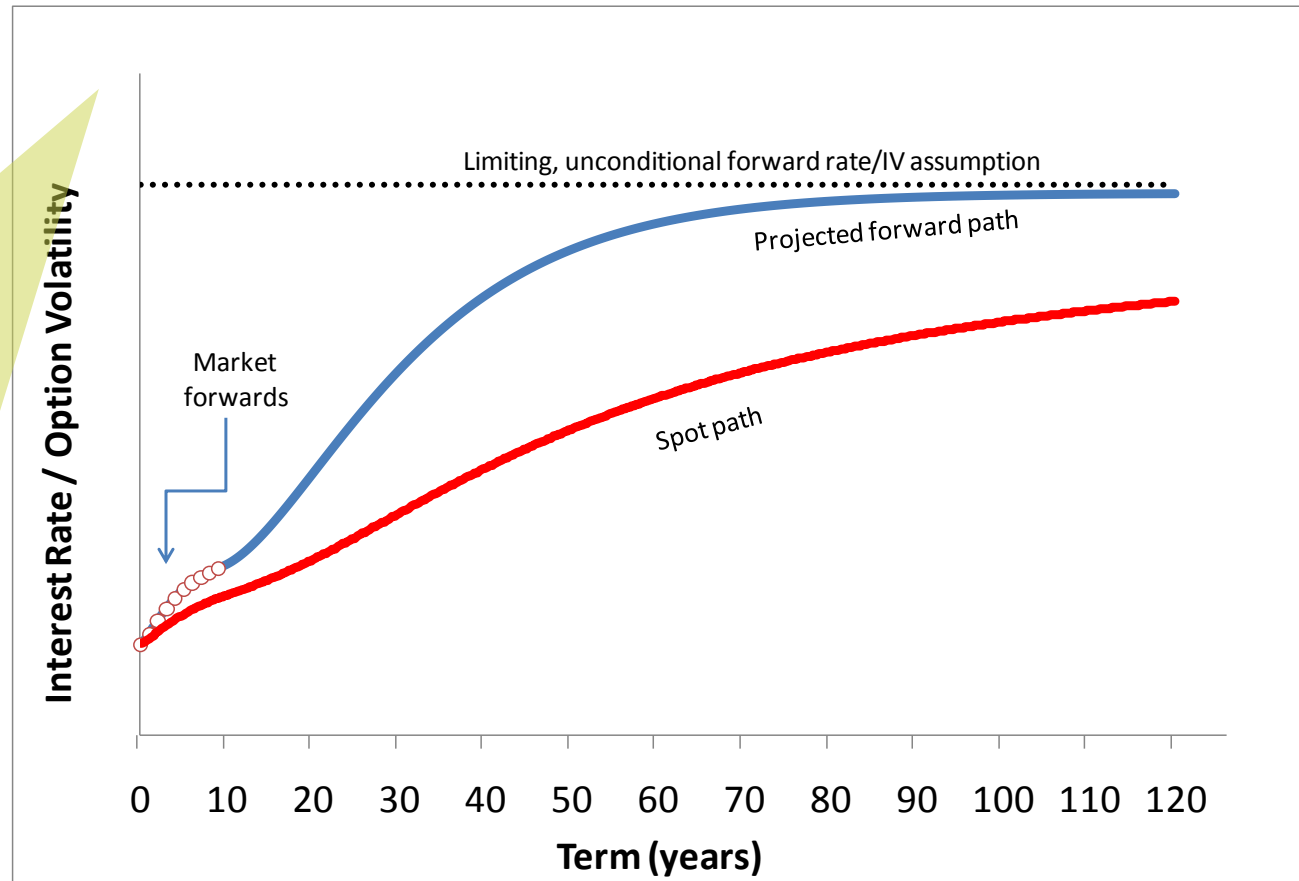
Longest maturity for extrapolation (years)	<i>Term of discount bond (years)</i>			
	30	50	70	100
30	21%	38%	58%	90%
20	22%	33%	45%	63%
10	27%	44%	61%	87%
5	33%	55%	77%	111%
1	48%	80%	112%	161%

# B+H Extrapolation principles:

## *The basic idea*

Extrapolation requires us to face three questions:

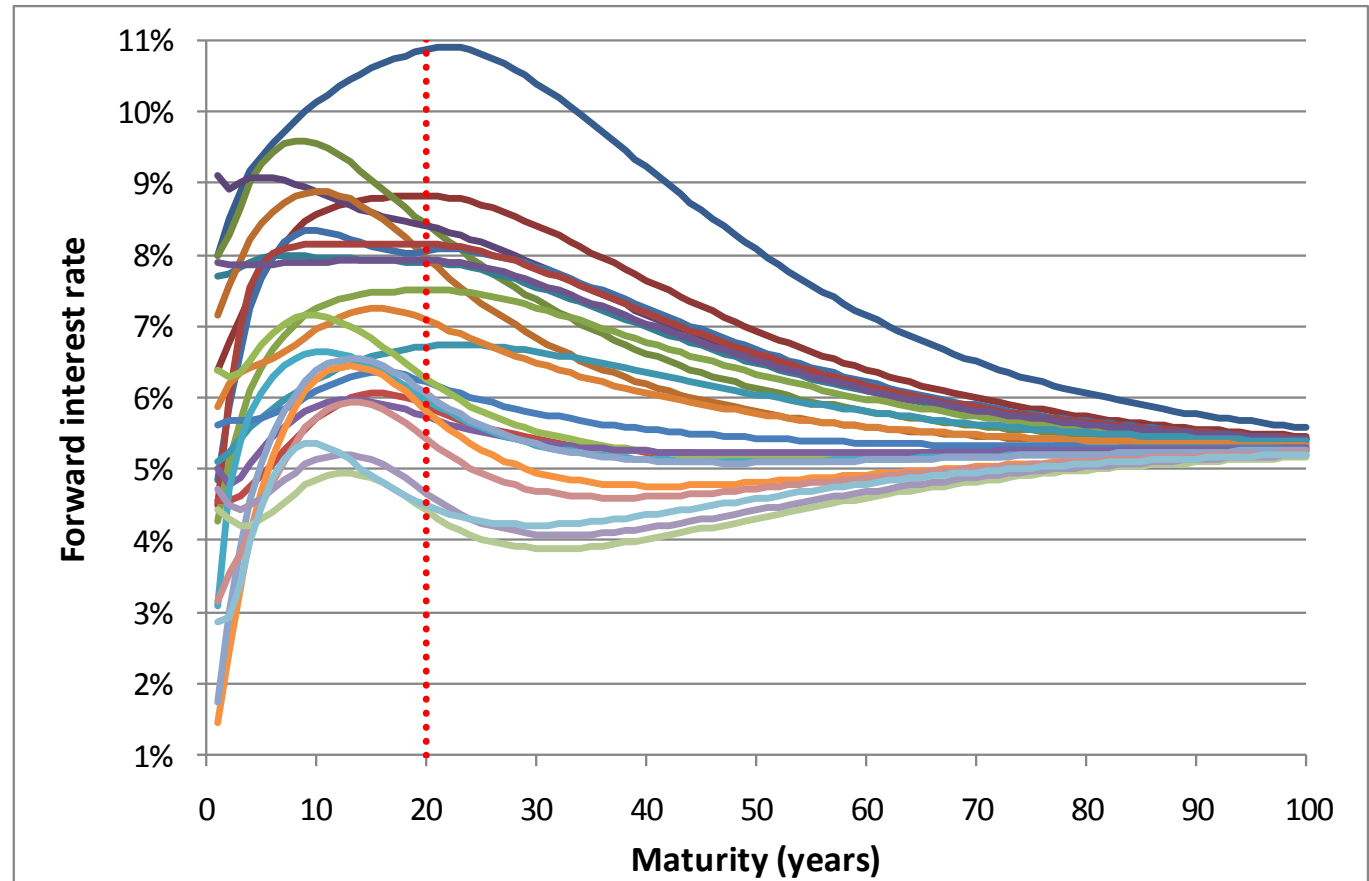
- 1) What is the longest market forward interest rate that we can observe?
- 2) For the purposes of extrapolation, what is an appropriate assumption for the very long-term 'unconditional' or 'limiting' forward rate?
- 3) What path should be set between the longest market rate and the unconditional forward rate?



# Interest rate extrapolation example

## USD (1985-2007)

- + Unconditional 'anchor' produces greater stability in mark-to-model ('level-3') valuations.



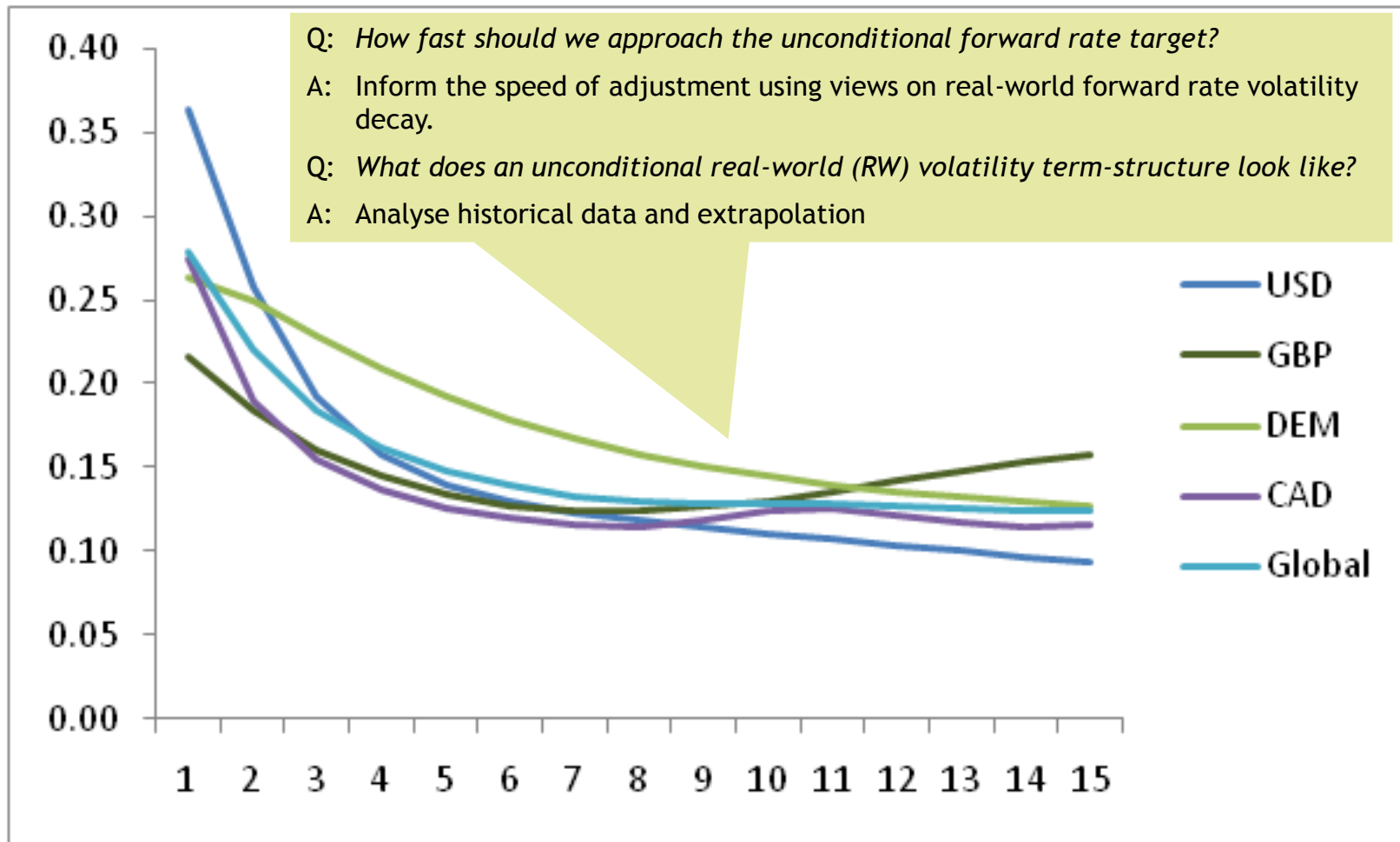


# Drivers of the long term forward rate

- + Market expectations of future interest rates
  - This involves setting a target for the unconditional expected short rate
- + Uncertainty about the future path
  - Investors' required risk premiums on bonds
  - Convexity adjustment

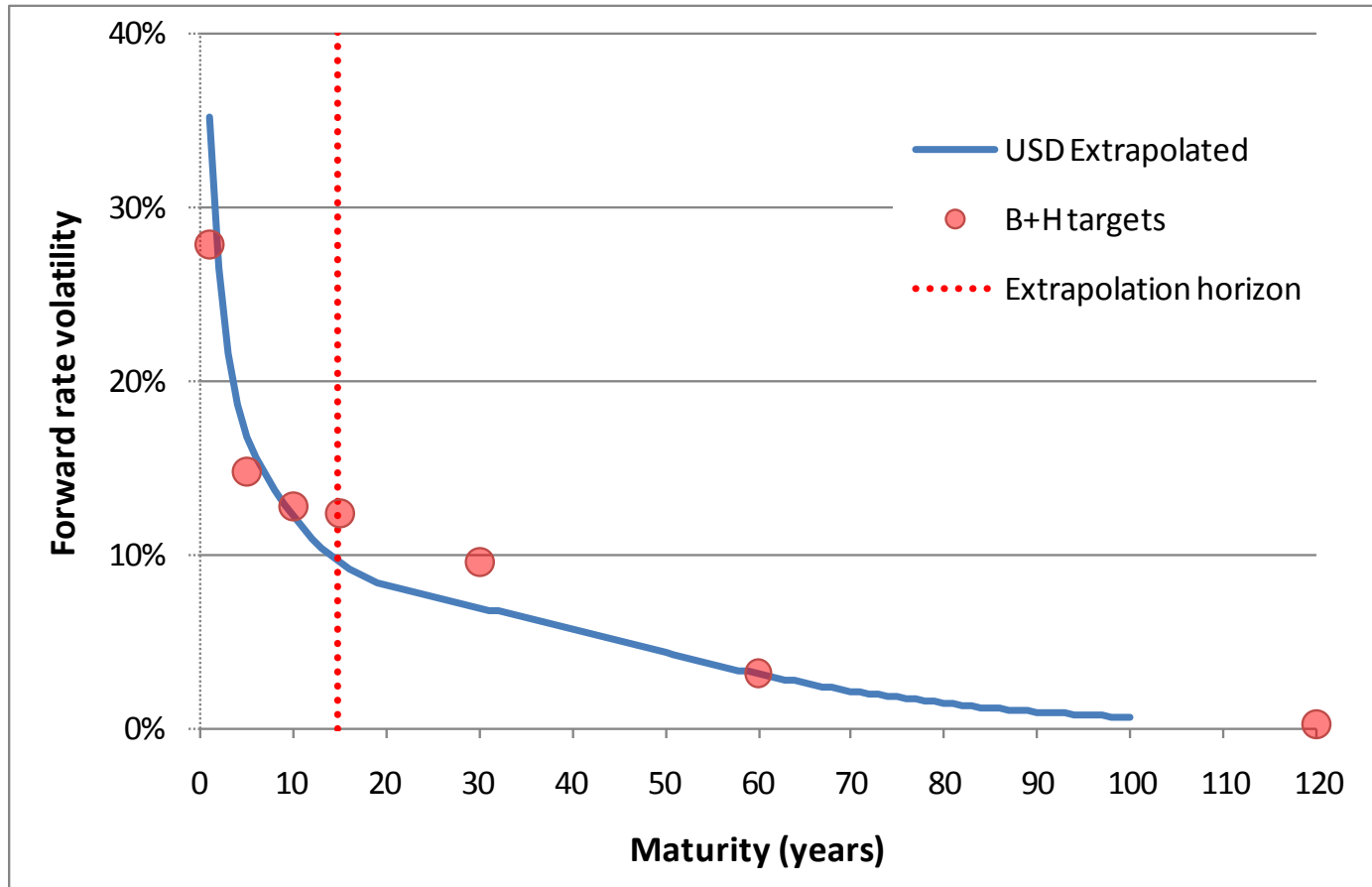
Assumptions for 2008	Government bond calibration	Swap calibration
Expected real interest rate	1.8%	2.0%
Expected inflation	2.4%	2.4%
Expected short rate	4.2%	4.4%
Term premium	1.5%	1.7%
Convexity adjustment	-0.4%	-0.4%
Unconditional forward rate	5.3%	5.7%

# Volatility estimated from the data



+ Volatility (sd) of changes in log nominal forward rates

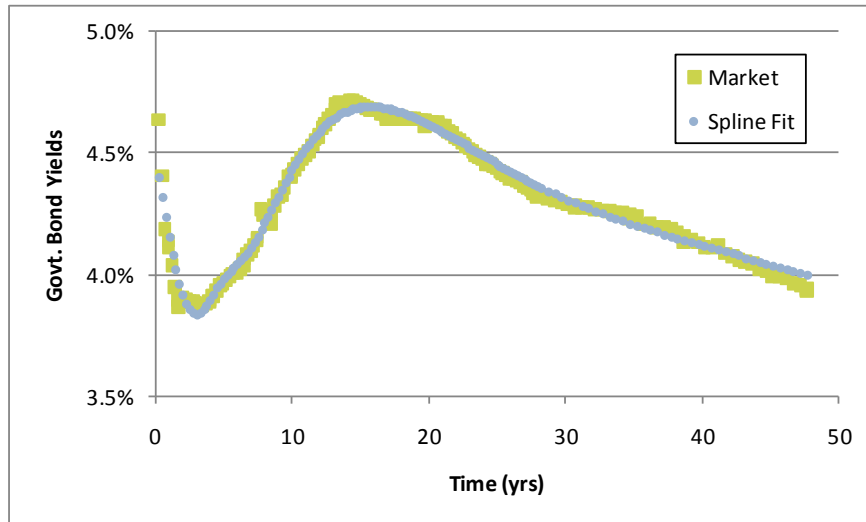
# The target volatility term structure



# Fitting the market:

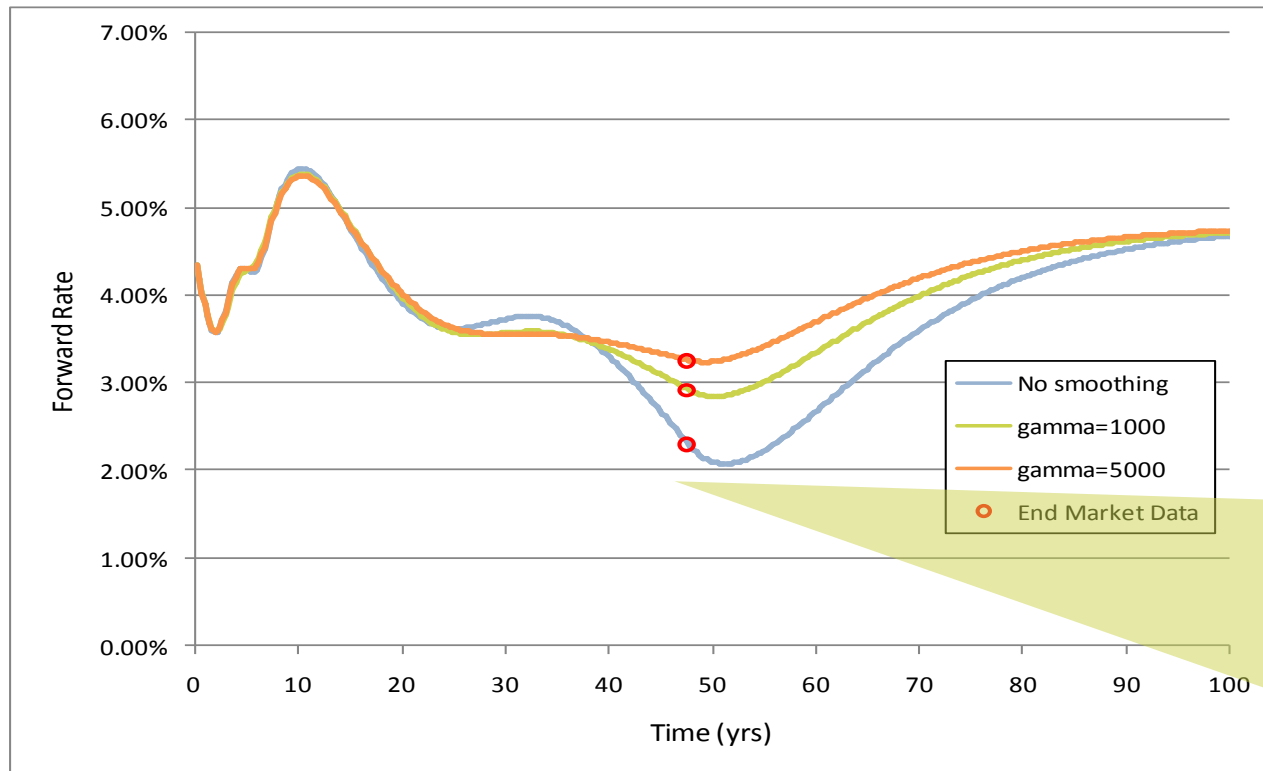
## Cubic Spline Interpolation

- + Standard parametric approaches (*Nelson-Siegel (1987), Svensson(1995)*)
  - Fail to fit complex shapes
  - Long-term forward rate influenced by the market
- + Specify curve as a cubic spline
  - Freedom to fit complex shapes e.g. GBP March 2008:



- Satisfies requirements for accuracy, continuity and smoothness.

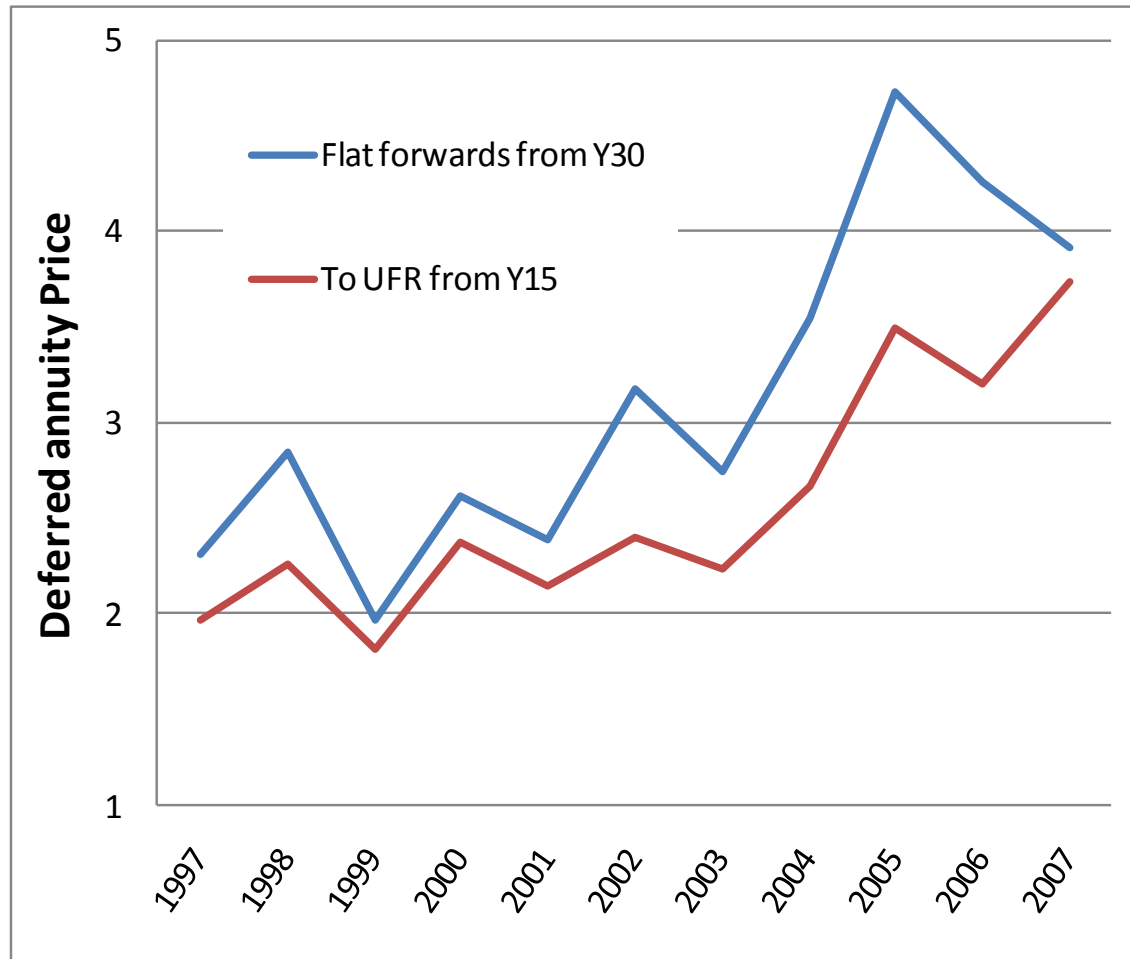
# Putting it all together: Controlling the extrapolation



Very small changes in long-dated market rates can have a large impact on the extrapolated curve:

- So, fit with a penalty function to minimise slope over last section of market data
- Means that the level and not (potentially spurious) slope of market forward curve is the primary driver for extrapolation.

# USD price of a 25-year annuity deferred for 30 years using alternative forward interest rate extrapolation methods



# CEIOPS' advice

## 34/09 'blue text' (November 2009)

3.74 CEIOPS recognises the importance of the choice of the extrapolation technique and thus does not prescribe the method for extrapolating the interest rate curve at this stage. Instead, during the Level 3 process, CEIOPS will develop a set of principles for the choice of an appropriate extrapolation method and will, based on these principles, choose for each currency the method deemed to be most appropriate

EU Commission response was to set up a task force to review CEIOPS' guidance on:

- + Illiquidity premium
- + Government bond or swap curve
- + Extrapolation

The task force's report was due on 31<sup>st</sup> January.

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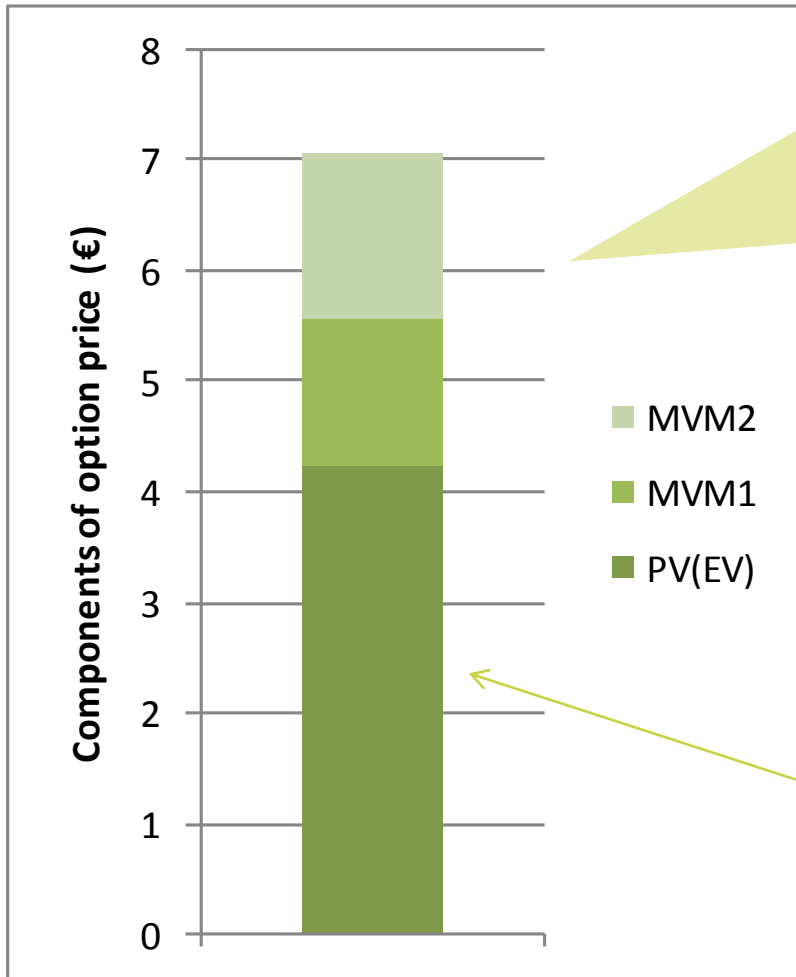
# Valuation of options & 'mark-to-model' of long-dated, untraded options



# Example: Unravelling option prices

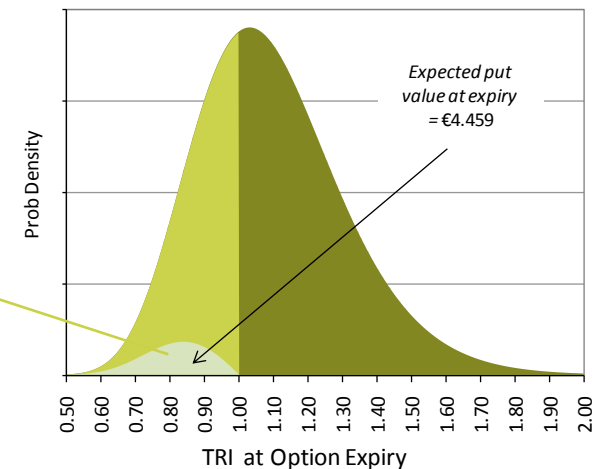
- + Suppose an insurer promises a total return on a €100 investment in line with an equity index over a 1-year period with a simple guarantee fixed also at €100.
- + The product can be replicated with a €100 investment in the underlying equity index and an at-the-money 1-year put option on the total return index.
- + The value of this portfolio tells us the fair value of the liability (irrespective of how the insurer chooses to invest the policyholder's premium).
- + The put option sells at a price of €7.081 (IV=24%) and the 1-year interest rate is 5% compounded continuously.
- + Suppose that we know, for certain, that the distribution for the end-of-year total return index is log normally distributed with a standard deviation of 20% pa and an arithmetic mean of 4% pa (compounding continuously) in excess of the risk-free rate of interest.

# Components of option put value



Option can be thought of as comprising 3 components:

1. PV of the option payoff.
2. MVM1: an adjustment for using the 'correct' discount rate (as in a BS world).
3. MVM2: the impact of allowing for the true dynamics of asset prices and the required returns (i.e. cost of capital) or traders



# Comparison of MC approach vs risk margin method

Strike	Market consistent value (MC)	Risk margin (RM)	Ratio (MC/RM)	Market-Implied CoC
0.60	0.059	0.006	9.8	Na
0.80	1.325	0.888	1.5	12%
1.00	7.081	5.140	1.4	19%
1.20	18.807	15.788	1.2	22%
1.40	34.739	31.580	1.1	21%

*The 99.5% required capital at the end of the period as €31.471 which requires a start-of-period SCR of €29.936. Assuming this required capital runs off evenly over the year, applying the 6% charging rate we can calculate a risk margin of €0.898.*

# Conclusions

- + For a market option which carries exposure to general market risk, the present value of best-estimate cash flows (using a risk-free rate) can turn out to be quite different to the ‘correct’ option value:
  - Adjust for the correction in the discount rate given the systematic exposure of the cash flows
  - and the hedging and capital costs of traders.
- + Option prices - along with all financial asset prices - already contain MVMs. Option models have the capability to allow for MVMs in an economically sensible way.
- + An option valuation model is the simplest means of producing economically coherent ‘pseudo-prices’ for missing maturities or strikes.
- + The use of models for marking non-hedgeable risks to market is by no means straightforward.

# Extrapolation of option IV or price surfaces

- + Additional complexity
  - Extrapolation is ideally required by both term and strike
  - Avoid arbitrage across the entire price/IV surface
- + This is only possible with a model which:
  - Provides a good (i.e. realistic) description of real-world asset and asset volatility behaviour
  - Can be adjusted to describe *pricing* of options (i.e. risk-neutral dynamics of asset and asset volatility)
- + ‘Stochastic Volatility / Jump Diffusion’ (SVJD) models are one class of models which offer this capability.
- + Extrapolation work must be integrated with views on the real-world volatility behaviour and dynamics of volatility changes.
  - Real-world expectations ‘anchor’ long-term extrapolated prices
- + Pricing adjustments compensate for what the model leaves behind:
  - ‘risk-neutral’ volatility is adjusted upwards (vs real-world)
  - ‘risk-neutral’ volatility shocks are more persistent (vs real-world)

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# Towards greater consistency

# Towards greater consistency?

- + We strongly believe - in common with accountants - that 'mark-to-model' is the only viable approach to market-consistent valuation of non-traded market-contingent cash flows.
- + In practice, all approaches require expert judgment (in model and parameter choice) and this creates the potential for divergence of results among firms (for the same market-contingent cash flow).
- + As a result, principles-based regulation on its own will not necessarily produce consistent results across firms without an additional overlay of guidance or prescription in key areas.

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Questions?



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