Systemic Risk

Colin Priest BEc MAppStat FIAA

18th August 2011

Agenda

1. Theory (mathematical but I hope not too scary!)
2. Estimation
3. Practical Applications
    1. Risk margins
    2. Diversified risk margins
    3. Capital modelling
    4. Strategic planning
    5. Portfolio management

Theory

Systemic Risk

- Risk that is shared by more than one exposure or more than one claim. Risk that affects more than one exposure or more than one claim
- “the element of the total coefficient of variation that is constant across the whole line of business, irrespective of the size of the liability" (Bateup and Reed, 2001)
- Examples: inflation, weather, natural catastrophe, insurance cycle etc

Non-Systemic (Independent) Risk

- Risk that is specific to only a single exposure or claim. Risk that only affects a single exposure or claim
- “the element of the total coefficient of variation that is related to the size of the liability" (Bateup and Reed, 2001)
- Examples: a tired bus driver falls asleep while driving and crashes, a doctor makes a mistake when treating a patient

Poisson or Negative-Binomial?

- For Poisson:
  - $E(k) = \lambda$
  - $Var(k) = \lambda$

- Have you ever noticed that your historical claim counts have a higher variance than you would expect?

Poisson or Negative-Binomial?

- What if λ isn't constant? If it varies from month to month.
- This could be due to environmental factors, such as weather conditions
- Then λ is a random variable too
- Variation in λ is systemic risk
- If λ is Gamma distributed, then the claim count follows a binomial distribution and $Var(k) > E(k)$

Binomial or Beta-Binomial?

- For Binomial lapse count, with lapse rate X:
  - $E(X) = p/n$
  - $Var(X) = pq/n$

- Have you ever noticed that your historical lapse rates have a higher variance than you would expect?

Binomial or Beta-Binomial?

- What if p isn't constant? If it varies from month to month.
- This could be due to seasonal factors, economic conditions, social networks etc
- Then p is a random variable too
- Variation in p is systemic risk
- If p is Beta distributed, then the number of lapses is Beta-Binomial distributed, and the lapse rate X follows a Beta distribution and
  $Var(X) > E(X) * (1 – E(X)) / n$
  i.e. the variance is greater than from a Binomial

Warning: Greek Letters

$$c_{ij} = \mu + \alpha\, \nu_i + \beta\, \varepsilon_{ij}$$

actual claim cost for j^th claim in group i → c_{ij}

average claim cost → μ

random variation affecting only group i → ν_i

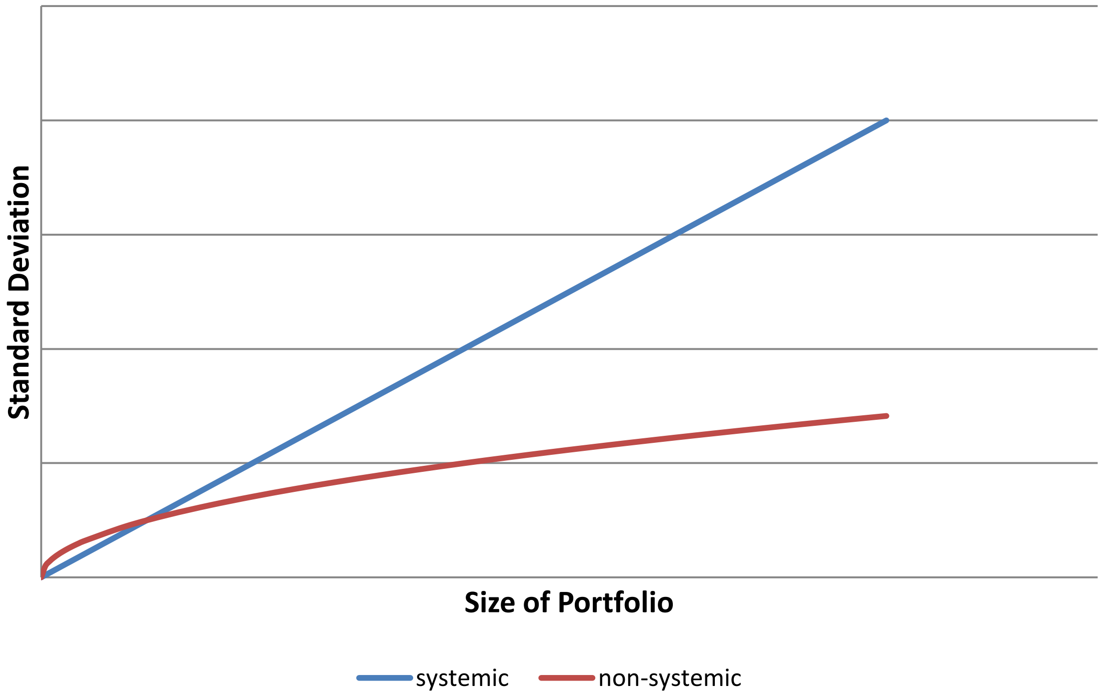
random variation affecting only the j^th claim in group i → ε_{ij}

- This is a “random effects” model

For a Portfolio of n Claims

$$Var(\Sigma\Sigma c_{ij}) = \alpha\Sigma n^2_i + \beta\Sigma n_i$$

$$SD(\Sigma\Sigma c_{ij}) = sqrt(\alpha\Sigma n^2_i + \beta\Sigma n_i)$$

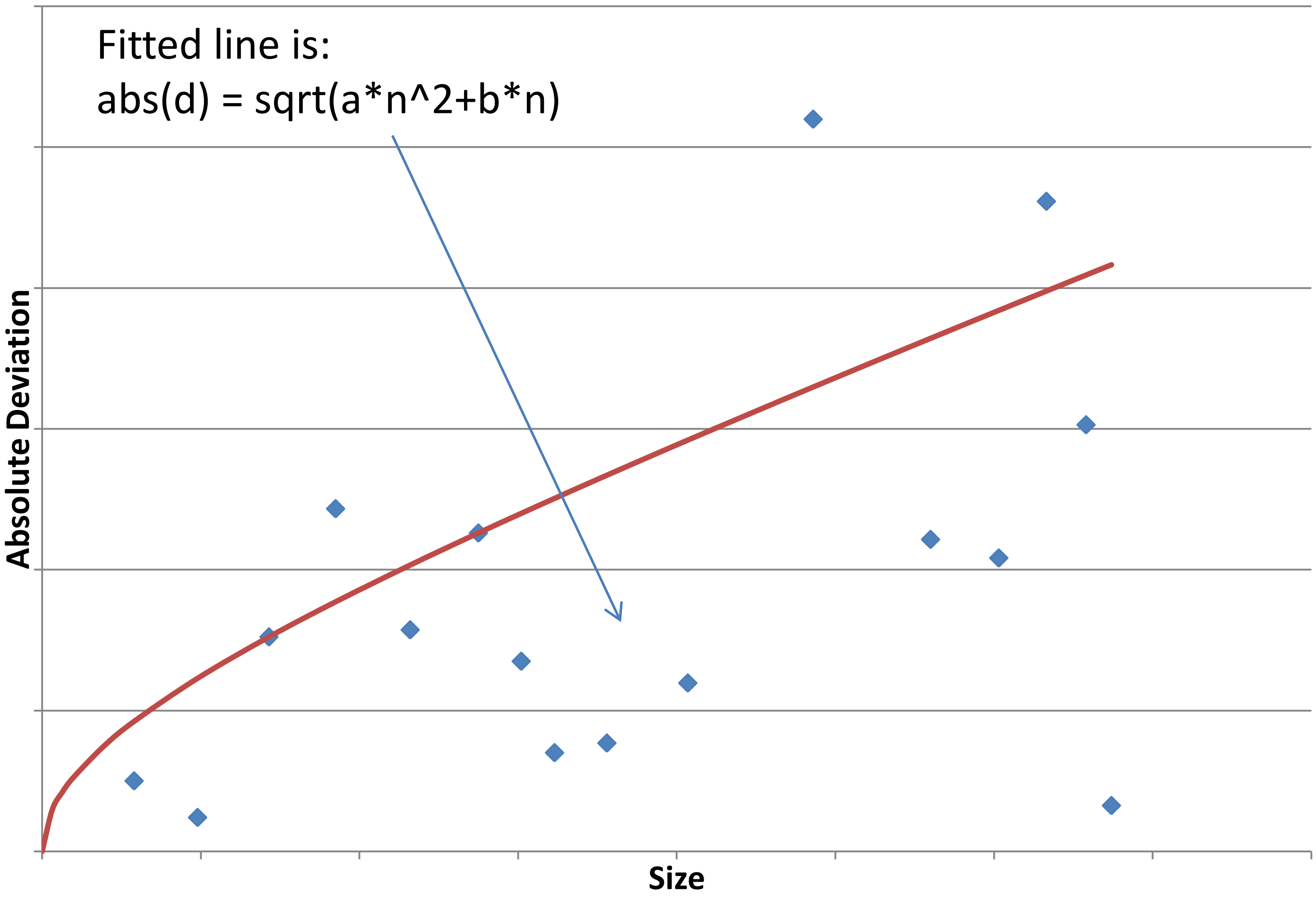


Estimation

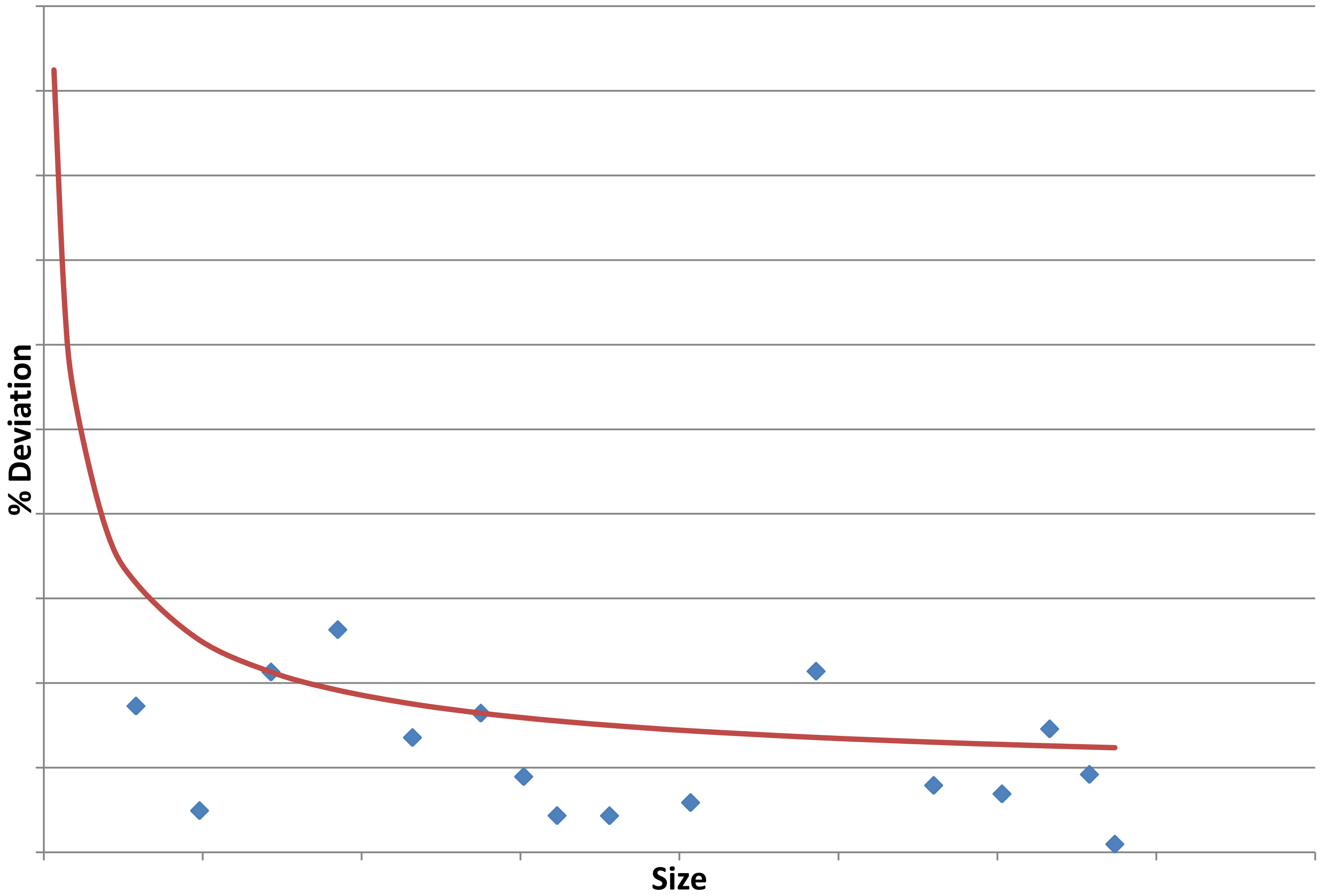
Estimation

1. Historical results
2. Data partitioning
3. Claims triangles

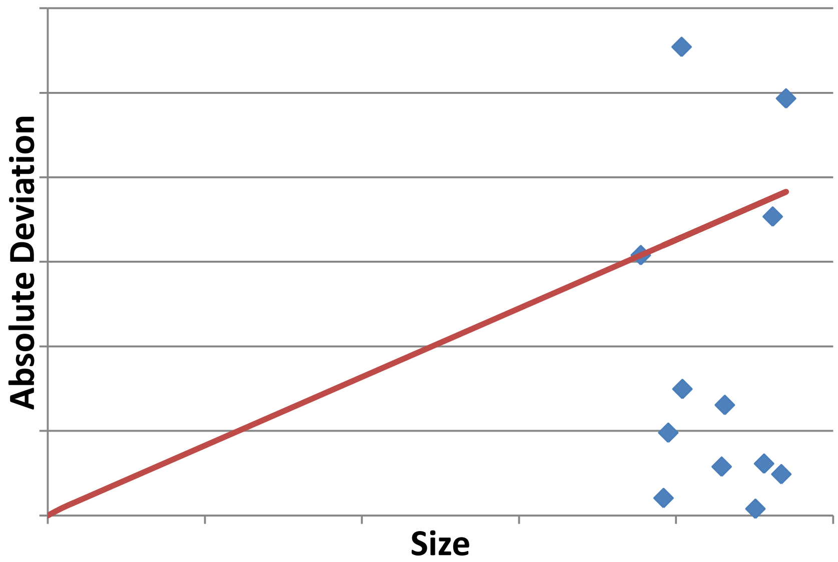
Historical Results



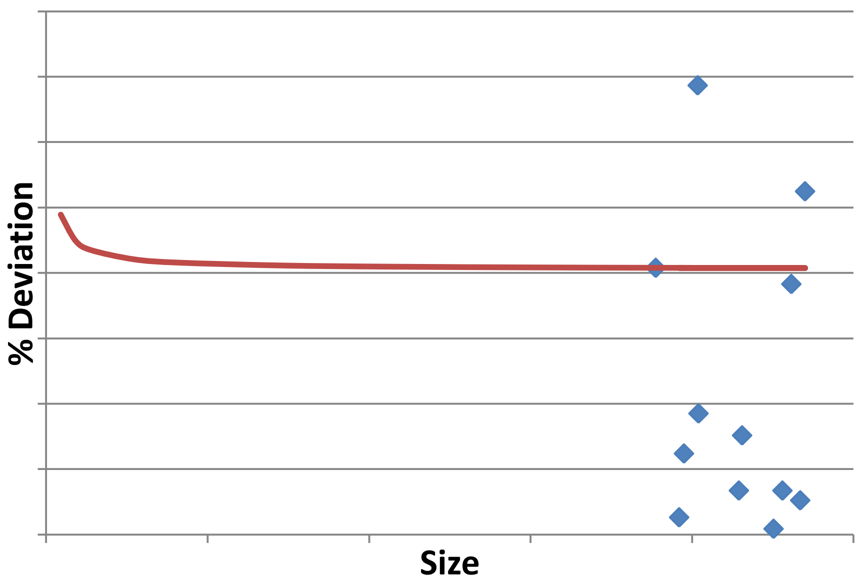
Historical Results



Poor Fit for Historical Results



- Data all clumped around the same size!

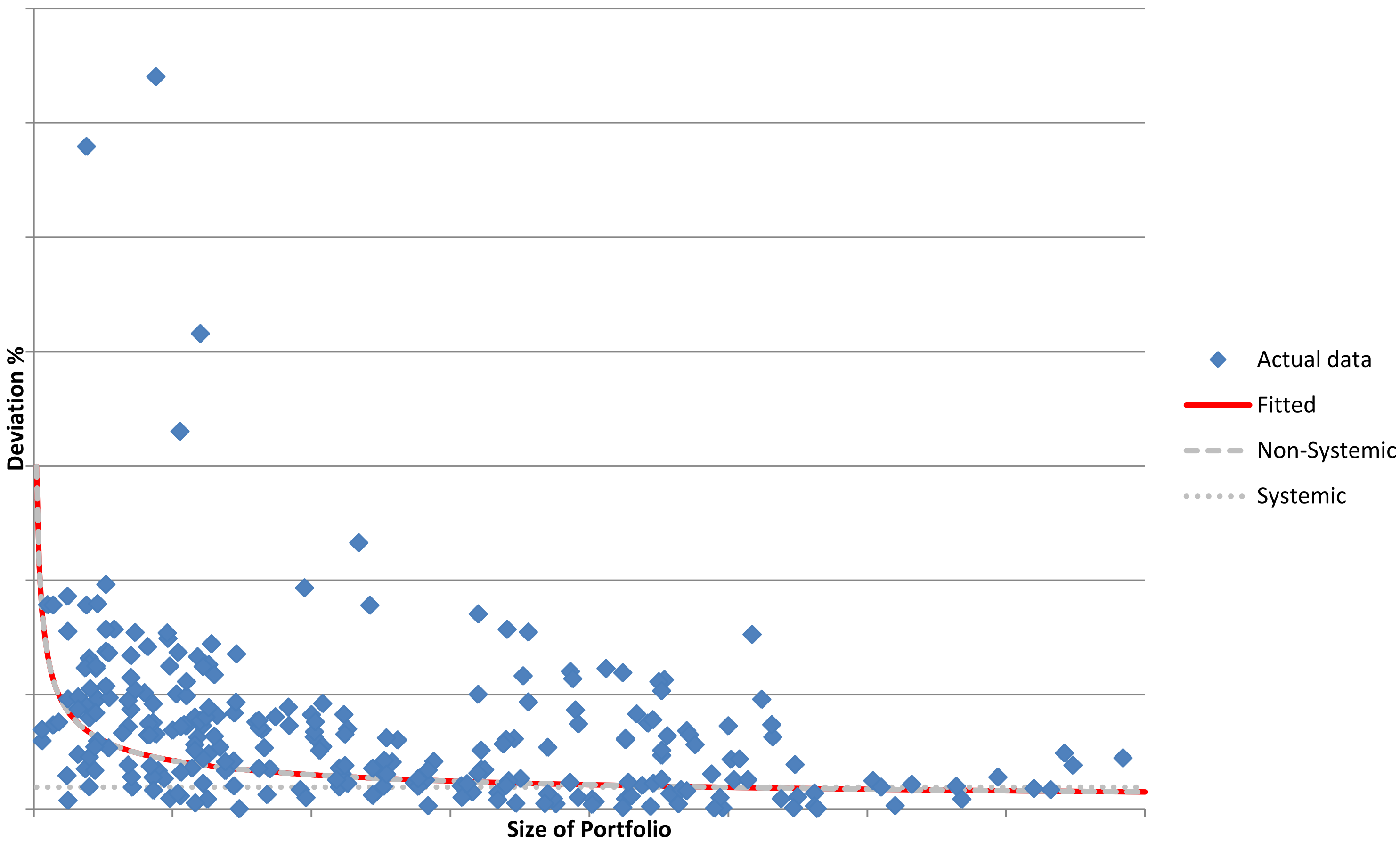


Data Partitioning

- Most insurers will have data like that shown in the previous slide
- Need more data points with small portfolio size

- Randomly (this is important!) split the existing data into smaller sub-portfolio data points.
- Repeated sampling of different sized sub-portfolios is OK.

Data Partitioning



Claims Triangles

- Systemic Risk along accident periods
  - Note the accident period effect

	Earned Premium	0	1	2	3	4	5	6
1	1,749,038	225,633	619,849	660,078	203,140	23,546	20,327	32,575
2	2,358,629	296,370	906,406	427,835	178,646	84,963	101,616	83,971
3	2,617,002	303,407	933,581	450,686	249,244	27,045	98,584	7,295
4	3,159,899	273,405	1,621,091	1,310,029	910,790	439,714	234,868	140,858
5	3,480,946	285,737	778,189	491,470	186,238	42,922	35,382	25,245
6	3,561,378	358,054	1,364,101	1,491,556	1,117,384	802,264	434,305	
7	3,703,335	349,259	868,717	1,166,016	426,749	400,139		
8	3,879,841	187,509	879,653	620,731	180,029			
9	3,857,307	237,752	917,759	1,014,382				
10	3,451,561	375,578	1,768,252					
11	3,774,444	993,321						

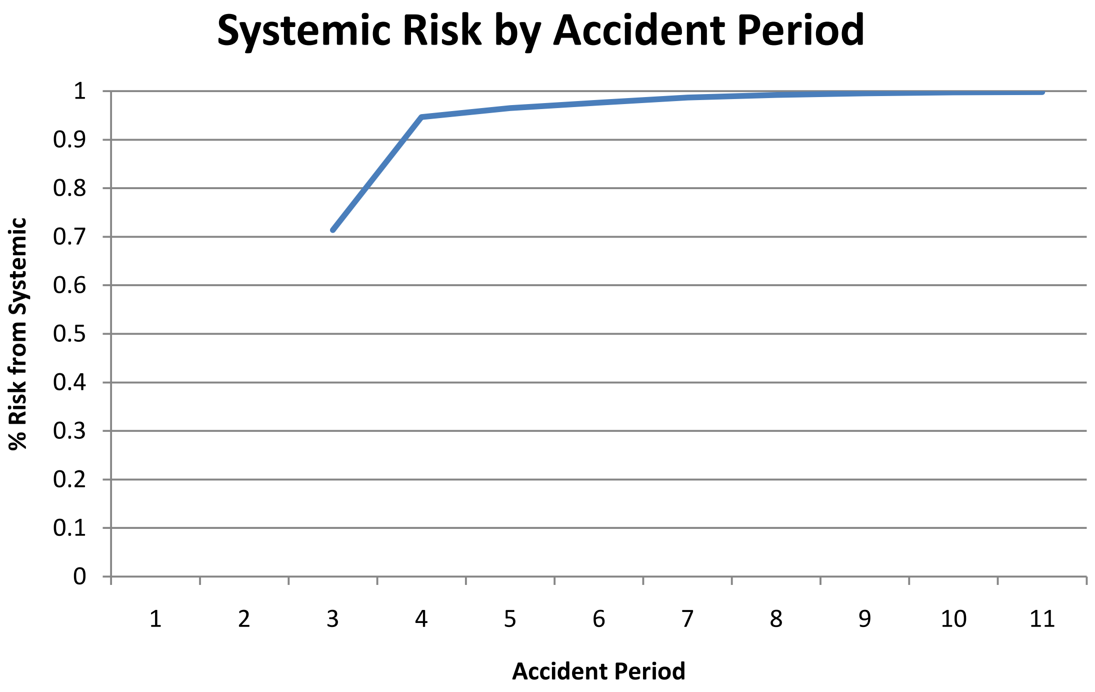
- Non-systemic risk varies by development period
  - Note the volatility of claims increases with development periods

Claims Triangles - Excel Tool

- Worked example using some anonymised data
  - simple model: no claim counts used
  - Estimates CoV of premium liability and correlation of OSC and premium liability

Outstanding Claims	
Central Estimate	8,652,772
CoV	32.5%
Risk Margin	23.8%
Premium Liability - Variance Estimate	
Central Estimate	2,075,944
CoV	36.9%
Risk Margin	27.2%
Correlation Coefficient Between Premium Liability and Outstanding Claim Liability	
Correlation	78%

Claims Triangles - Excel Tool



- Systemic risk dominates OSC for the latest accident periods
- Non-systemic risk increases in importance for older accident periods and the number of outstanding claims gets small – individual claim outcomes become more important to the uncertainty

Practical Uses

Practical Use 1: Risk Margins

- Growing portfolios
  - Risk changes with size of portfolio
  - Existing risk margin methods (e.g. Mack) don’t adjust for growth
  - If the portfolio is growing, then existing risk margin methods overestimate the required risk margin
- Accident period effects
  - Existing risk margin methods (e.g. Mack) don’t adjust for high or low claim frequencies in an accident period
  - So existing risk margin methods may overestimate or underestimate the required risk margin

Practical Use 1: Risk Margins

- Premium liabilities
  - CoV of PL is greater than CoV of historical loss ratios
  - A better estimate is:

$$\hat{\sigma}_P^2 = \frac{\alpha^2 n_P^2 + \beta^2 n_P}{\alpha^2 n_A^2 + \beta^2 n_A} \left(\frac{1}{n} \sum_i \left(\left[\hat{D}_i \right]^2 + \hat{\sigma}_i^2 \right) - \left(\frac{1}{n} \sum_i \hat{D}_i \right)^2 \right)$$

$$\begin{aligned}
\hat{D}_i &= \sum_{j=1}^{n-i} \sum_k c_{ijk} + z_i \bar{\mu} + \frac{z_i}{x_i} \sum_{j=1}^{n-i} \sum_k \left(c_{ijk} - \bar{\mu} \right), \\
\text{where } \bar{\mu} &= \frac{1}{w} \sum_{i=1}^{n} \sum_{j=1}^{n-i} \sum_k c_{ijk} \\
\text{and } x_i &= \sum_{j=1}^{n-i} n_{ij} \text{ and } z_i = \sum_{j=n-i+1}^{n} n_{ij} \text{ and } w = \sum_{i=1}^{n} \sum_{j=1}^{n-i} n_{ij}. \\
\hat{o}_i &= \frac{z_i}{x_i} \sum_{j=1}^{n-i} \sum_k c_{ijk}.
\end{aligned}$$

Use 2: Diversified Risk Margins

- Maximum possible correlation

- Correlation between OSC & PL

$$Corr(\hat{o},\hat{p}) = \frac{\sum_{i=1}^{n} z_i (x_i + z_i)\left(\alpha^2 + \frac{\beta^2}{x_i}\right)}{\sqrt{\left(\sum_{i=1}^{n} z_i^2 \left(\alpha^2 + \frac{\beta^2}{x_i}\right)\right) \times \left(\sum_{i=1}^{n} (x_i + z_i)^2 \left(\alpha^2 + \frac{\beta^2}{x_i}\right)\right)}} \times \frac{\alpha^2 n_P n_A}{\sqrt{\alpha^2 n_P^2 + \beta^2 n_P}\sqrt{\alpha^2 n_A^2 + \beta^2 n_A}}$$

$$\text{where } x_i = \sum_{j=1}^{n-i} n_{ij} \text{ and } z_i = \sum_{j=n-i+1}^{n} n_{ij}$$

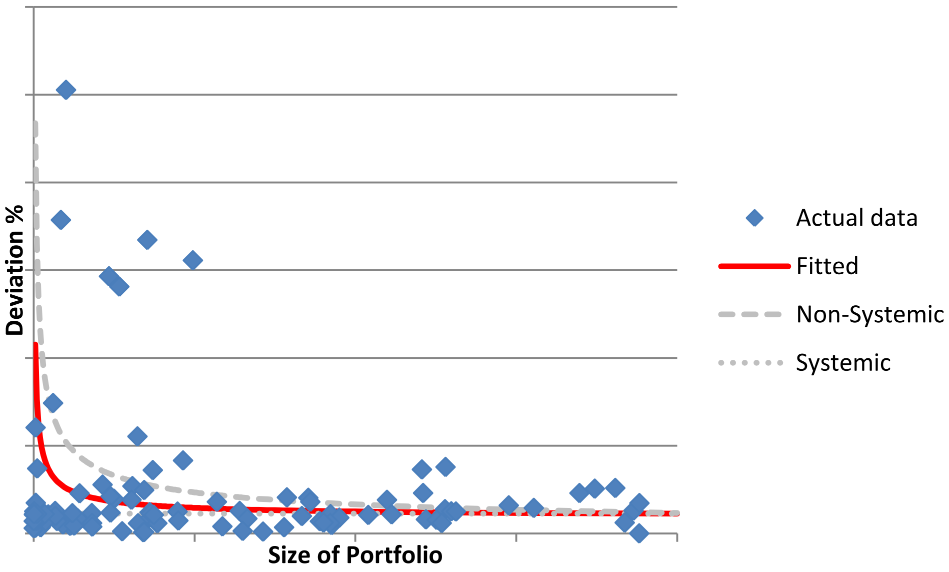
Use 3: Capital Modelling

- Avoid implicit assumption that cohorts are independent
  - This implicit assumption will underestimate capital requirements

- Cat event counts are not Poisson distributed
  - Weather – see Australian claims in 2010/2011
  - Earthquake – NZ claims in 2010/2011
  - This implicit assumption will underestimate capital requirements

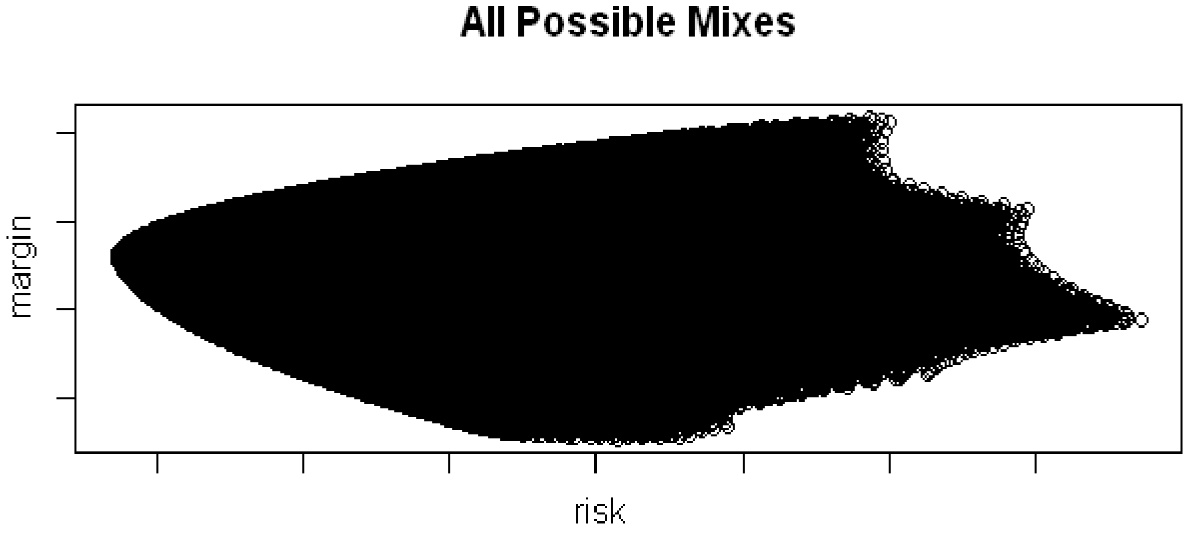
Use 4: Strategic Planning

- Estimate capital efficiency from growing a portfolio
  - Is growth the best way to improve capital efficiency?
  - Which lines of business to grow?



Use 5: Portfolio Management

- Optimal mix of business
  - Risk changes with size of sub-portfolio
  - Correlation changes with size of sub-portfolio



Want to Know More?

- Download link:
  - Research paper
  - Excel file

https://public.me.com/colinpriest/SAS-talk-18Aug2011

The Singapore Actuarial Society will also upload copies of the files at their web site: http://www.actuaries.org.sg/