Implementing Stochastic Claims Reserving in Spreadsheet

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Mack method – MSE(R)

$$MSE(R) = \sum_{i=2}^{n} \{MSE(R_i) + covariance\}$$

$$MSE(R_i) = C_{i,n}^2 \sum_{k=n+1-i}^{n-1} \left\{ \frac{\alpha_k^2}{f_k^2} \left(\frac{1}{C_{i,k}} + \frac{1}{\sum_{j=1}^{n-k} C_{j,k}} \right) \right\}$$

MSE(R_i) of reserve R for accident year *i* which can be broken down into parameter variance and process variance

Mack method – alpha

For k=1 to k=n-2

$$\alpha_k^2 = \frac{1}{n-1-k}\sum_{j=1}^{n-k} C_{j,k}\left(\frac{C_{j,k+1}}{C_{j,k}} - f_k\right)^2 = \frac{1}{n-1-k}\sum_{j=1}^{n-k} D_{j,k}$$

For k=n-1

$$\alpha_{n-1}^2 = \min\left(\frac{\alpha_{n-2}^4}{\alpha_{n-3}^2}, \min(\alpha_{n-3}^2, \alpha_{n-2}^2)\right)$$

Mack method – covariance

$$covariance_i = C_{i,n}\left(\sum_{j=i+1}^{n} C_{j,n}\right)\sum_{k=n+1-i}^{n-1}\frac{2\alpha_k^2\ /\ f_k^2}{\sum_{n=1}^{n-k} C_{n,k}}$$

The formula above calculates the covariance for accident year *i*

Mack method – recursive method

There are a total of 9 steps in the recursive method which can be seen as comprising 3 major parts:

- Firstly, estimate the Chain Ladder mean reserve by finding f_k which are used to determine future claim $C_{i,k}$
- Next, estimate the MSE of reserve for each accident year by determining α_k^2
- Finally, estimate covariance by using α_k^2, f_k and $C_{i,k}$ found earlier

Mack method – data

Historical loss development study (1991) Automatic Facultative General Liability data (excluding asbestos and environmental) from 1981 (i=1) to 1990 (i=10) where cumulative loss was expressed in $’000

	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10
i=1	5,012	8,269	10,907	11,805	13,539	16,181	18,009	18,608	18,662	18,834
i=2	106	4,285	5,396	10,666	13,782	15,599	15,496	16,169	16,704	
i=3	3,410	8,992	13,873	16,141	18,735	22,214	22,863	23,466		
i=4	5,655	11,555	15,766	21,266	23,425	26,083	27,067			
i=5	1,092	9,565	15,836	22,169	25,955	26,180				
i=6	1,513	6,445	11,702	12,935	15,852					
i=7	557	4,020	10,946	12,314						
i=8	1,351	6,947	13,112							
i=9	3,133	5,395								
i=10	2,063									

Mack method – definition

$C_{i,k}$ = accumulated total claims amount for accident year *i* and development year *k*

$R_i = C_{i,10} - C_{i,11-i}$ = claims reserve for accident year *i*

$R = R_2 + R_3 + \ldots + R_{10}$ = total claims reserve (all accident years)

f_k = age-to-age factor for development year *k*

Mack method – step 0

Estimate mean reserve

Step 0 (a) : Calculate age-to-age factors f_k for k=1 to 9

$$f_k = \frac{\sum_{j=1}^{10-k} C_{j,k+1}}{\sum_{j=1}^{10-k} C_{j,k}}\,, \qquad 1 \le k \le 9$$

Step 0 (b) : Calculate estimate of future $C_{i,k}$ for i=2 to 10 and k=2 to 10

For i=2 to 10 and k=1 to 9, $C_{i,k+1} = f_k * C_{i,k}$

Mack method – step 1

Step 1 : Intermediate step to calculate α_k^2

Calculate $D_{i,k}$ for i=1 to 9 and k=1 to 8

$$D_{j,k} = C_{j,k}\left(\frac{C_{j,k+1}}{C_{j,k}} - f_k\right)^2$$

Mack method – step 2

Step 2 (a) : Calculate α_k^2 for k=1 to 8 recursively backwards starting from k=8

$$\alpha_k^2 = \frac{1}{9-k}\sum_{j=1}^{10-k} C_{j,k}\left(\frac{C_{j,k+1}}{C_{j,k}} - f_k\right)^2 = \frac{1}{9-k}\sum_{j=1}^{10-k} D_{j,k}$$

Step 2 (b) : Calculate α_9^2

$$\alpha_9^2 = \min\left(\frac{\alpha_8^4}{\alpha_7^2}, \min(\alpha_7^2, \alpha_8^2)\right)$$

Mack method – step 3

Step 3 : Calculate the denominator of the last term of parameter variance for k=1 to 9

$$\sum_{j=1}^{10-k} C_{j,k}$$

Mack method – step 4

Step 4 (a) : Calculate last term of MSE of accumulated claim reserve for each future *i*=2 to 10 and *k*=1 to 9 using the formula:

$$\frac{\alpha_k^2}{f_k^2}\left(\frac{1}{C_{i,k}}+\frac{1}{\sum_{j=1}^{10-k} C_{j,k}}\right)$$

Step 4 (b) : Calculate square root of MSE(R_i) for *i*=2 to 10 by multiplying the ultimate claim $C_{i,10}$ estimate and the sum of the results in Step 4(a) from column *k*=1 to 9

$$C_{i,10}^2 \sum_{k=11-i}^{9}\left\{\frac{\alpha_k^2}{f_k^2}\left(\frac{1}{C_{i,k}}+\frac{1}{\sum_{j=1}^{10-k} C_{j,k}}\right)\right\}$$

Mack method – step 5

Step 5 : Calculate the middle term of covariance for *i*=2 to 9

$$\sum_{j=i+1}^{10} C_{j,10}$$

Mack method – step 6

Step 6 : Calculate the last term of covariance for development years *k*=1 to 9 using result from Step 0, Step 2 and Step 3

$$\frac{2\alpha_k^2 \,/\, f_k^2}{\sum_{n=1}^{10-k} C_{n,k}}$$

Mack method – step 7

Step 7 : Calculate the sum of the last term of covariance for accident year *i*=2 to 10 recursively backwards from step 6 starting from development year 9

$$\sum_{k=11-i}^{9} \frac{2\alpha_k^2 \,/\, f_k^2}{\sum_{n=1}^{10-k} C_{n,k}}$$

Mack method – step 8

Step 8 : Calculate MSE(R_i) plus covariance for i=2 to 10 where covariance is calculated as ultimate accumulated claim $C_{i,10}$ multiply by the result of Step 5 multiply by the result of Step 7

$$Covariance = C_{i,10}\left(\sum_{j=i+1}^{10} C_{j,10}\right)\sum_{k=11-i}^{9}\frac{2\alpha_k^2\ /\ f_k^2}{\sum_{n=1}^{10-k} C_{n,k}}$$

Mack method – lognormal assumption

$$E(R) = e^{\mu+\frac{\sigma^2}{2}} = 52{,}135$$

$$MSE(R) = e^{2\mu+\sigma^2}\left(e^{\sigma^2} - 1\right) = 724{,}094$$

$$\sigma^2 = ln\left(1 + \frac{MSE(R)}{E(R)^2}\right) = 0.236178$$

$$\mu = ln(R) - \frac{\sigma^2}{2} = 10.743507$$

$$At\ 90\%\ percentile, z = 1.2816, e^{\mu+z\sigma} = 86{,}363$$

Mack method – references

<u>Reference</u>

- Mack T. 1993, "Measuring The Variability of Chain Ladder Reserve Estimates", CAS meeting May 1993

- Mack T., "crm2-D6spreadsheet.xls", Institute of Actuaries Claim Reserve Manual Volume 2 Section D6

ODP Model – Claims Reserving

Estimates of future payment – same outcome as Chain Ladder

$$\hat{C}_{i,j} = \widehat{m}_{i,j} = exp\left(\hat{\eta}_{i,j}\right)$$

Linear predictor estimated by MLE method implemented in Microsoft Excel Solver

$$\eta_{i,j} = c + \alpha_i + \beta_j$$

Mean Square Error of Prediction for each accident year

$$MSEP\left[\hat{C}_{i,j}\right] \approx \ \phi\, \widehat{m}_{i,j} + \widehat{m}_{i,j}^{2}\, Var\left[\hat{\eta}_{i,j}\right]$$

ODP Model – Claims Reserving

Toggle between PowerPoint and Excel to demonstrate the 14 steps to calculate MSEP for each accident year.

1. Calculate age-to-age factors then project future claim paid

2. Set up arbitrary initial values for all parameters

3. Find value of linear predictor $\eta_{i,j} = c + \alpha_i + \beta_j$

4. Calculate mean $\mu_{i,j} = \exp(\eta_{i,j})$

5. Calculate log likelihood = $c_{i,j} * \ln(\mu_{i,j}) - \mu_{i,j}$

6. Find the values of the parameters that maximizes the log likelihood using Excel Solver.

ODP Model – Claims Reserving

7. Calculate (actual claim – expected claim)^2/expected claim then compute Pearson scale parameter

8. Set up the $y_{i,j}$ vector and X matrix from data

9. Calculate $z_{i,j} = (y_{i,j} - \mu_{i,j})/\mu_{i,j}$

10. Calculate W matrix

11. Compute $X^T WX$

12. Compute $(X^T WX)^{-1}$

13. Calculate the variance of the linear predictor.

14. Calculate MSEP, standard error and CV for each accident year

ODP Model – Claims Reserving

<u>Reference</u>

- England, P.D. and Verrall, R.J., 2002, “Stochastic Claims Reserving In General Insurance”, B.A.J. 8, III, 443-544

Bootstrap on ODP model

Toggle between PowerPoint to demonstrate the 14 steps to calculate MSEP for each accident year.

1. Calculate age-to-age factors and LDG

2. Calculate fitted cumulative loss payment

3. Calculate fitted incremental payment

4. Calculate incremental Chi-Square terms

5. Calculate residuals

6. Resample residuals

7. Calculate “Pseudo-Triangle” (what if) of cumulative loss payments

Bootstrap on ODP model

8. Store Simulation Output by running macro

9. Calculate mean, percentile, SD and CV

Bootstrap on ODP model

<u>Reference</u>

- England, P.D. and Verrall, R.J., 2002, “Stochastic Claims Reserving In General Insurance”, B.A.J. 8, III, 443-544

Thank you

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