Monitoring of Loss Emergence

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Introduction to Monitoring of Loss Emergence (1)

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- Claims liabilities usually form a huge amount of the liabilities in the balance sheet of an insurance company. In general, they are a multiple of the annual profit.
- Claims liabilities are estimates which are based on different actuarial techniques (e.g. standard reserving methods, survival ratio considerations etc.).
- The estimates rely upon specific assumptions (model assumptions, assumptions for adjustments of modeling results) which have to be checked for reasonability by the responsible reserving actuary.
- Even if they are reasonable at the point in time when the analysis is performed, the loss development can change (e.g. in the course of the following year) and with that the estimates might not be appropriate any longer.

Introduction to Monitoring of Loss Emergence (2)

- A full reserve study can usually be performed only once a year. This process might not be suitable, to timely react to new information.

- The task of **the monitoring process in reserving** is to early identify critical loss and reserve developments in individual segments and/or in the whole portfolio and to facilitate an appropriate reaction to these.

- Efficient techniques to identify critical reserve developments in the course of a year are essential.

Introduction to Monitoring of Loss Emergence (3)

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- The reserve monitoring can be based on various techniques. The selection which techniques are used depend upon
  - the underlying portfolio
  - the applied methods in the reserve study
  - necessary frequency of the monitoring process
  - regulatory requirements
  - other considerations (e.g. market studies, comparability with competitors)

Introduction to Monitoring of Loss Emergence (4)

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- In general, reserving departments are calculating the claims liabilities in third or fourth quarter for the year end.
- In quarters Q1, Q2 and Q3 of the next calendar year, new calculations are not performed, but the last assessment is used and is modified if necessary.
- Necessary modifications are based on:
  - observed (booked) claims development
  - information not yet visible in booked data, provided by
    - ➢ Claims department
    - ➢ Controlling department
    - ➢ Underwriting
    - ➢ Market developments/studies

METHODOLOGICAL PERSPECTIVE

Recap: Chain-Ladder model

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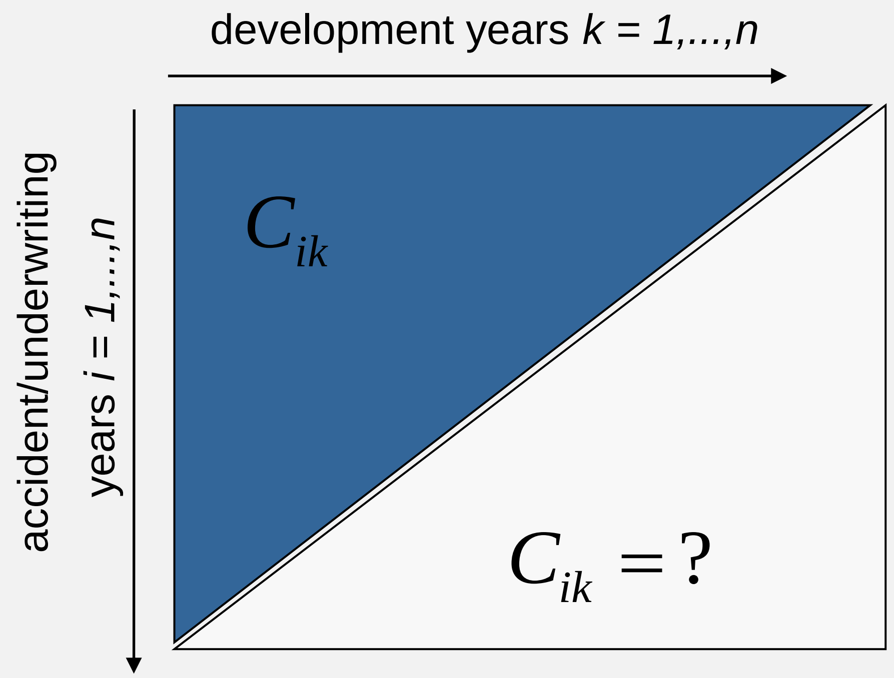
Notations in the run-off triangle of cumulative loss amounts (paid or incurred/reported), known for $i+k \leq n+1$:



Individual development factor is denoted by $F_{ik} = \frac{C_{ik}}{C_{i,k-1}}$

Recap: Chain-Ladder model

The Chain-Ladder (CL) model assumptions are:

(CL1) $\mathrm{E}(F_{ik} \mid C_{i1},\ldots,C_{i,k-1}) = f_k \quad , \; 1 \le i \le n \; , \; 2 \le k \le n$

(CL2) The accident years $\{C_{i1},\ldots,C_{in}\}$, $1 \le i \le n$, are independent

(CL3) $\mathrm{Var}(F_{ik} \mid C_{i1},\ldots,C_{i,k-1}) = \dfrac{\sigma_k^2}{C_{i,k-1}} \quad , \; 1 \le i \le n \; , \; 2 \le k \le n$

- This multiplicative model consists of *n-1* sub-models.
- CL1: The conditional expectation of the individual factors does not depend on the accident year.
- CL2: The independence condition is the basis of many „CL-propositions“, but may be violated in practice.
- CL3: The conditional variance of individual factors depends on the proceeding loss volume.

Recap: Chain-Ladder model

The basic Chain-Ladder estimators are:

$$\hat{f}_k := \sum_{i=1}^{n+1-k} w_i F_{ik} \quad \text{with } w_i := \frac{C_{i,k-1}}{\sum_{j=1}^{n+1-k} C_{j,k-1}}, \quad 2 \le k \le n$$

and

$$\hat{\sigma}_k^{\,2} := \frac{1}{n-k} \sum_{i=1}^{n+1-k} C_{i,k-1} \left(F_{ik} - \hat{f}_k \right)^2, \quad 2 \le k \le n-1$$

for development factors (age-to-age factors, link ratios) and variance parameters.

Recap: Chain-Ladder model

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Random error $\mathrm{Var}(C_{ik})$ (process risk) and estimation error $\mathrm{Var}(\hat{C}_{ik})$ (parameter error) are estimated recursively:

Random error:

$$\hat{\mathrm{Var}}(C_{ik}) = \begin{cases} 0 & , \; k \le n+1\text{-}i \\ \hat{\mathrm{Var}}(C_{i,k-1}) \cdot \hat{f}_k^{\,2} + \hat{C}_{i,k-1} \cdot \hat{\sigma}_{\mathrm{k}}^2 & , \; n+1\text{-}i < k \le n \end{cases}$$

Estimation error:

$$\hat{\mathrm{Var}}(\hat{C}_{ik}) = \begin{cases} 0 & , \; k \le n+1\text{-}i \\ \hat{\mathrm{Var}}(\hat{C}_{i,k-1}) \cdot \hat{f}_k^{\,2} + \dfrac{\hat{C}_{i,k-1}^2}{\sum_{j=1}^{n+1-k} C_{j,k-1}} \cdot \hat{\sigma}_{\mathrm{k}}^2 & , \; n+1\text{-}i < k \le n \end{cases}$$

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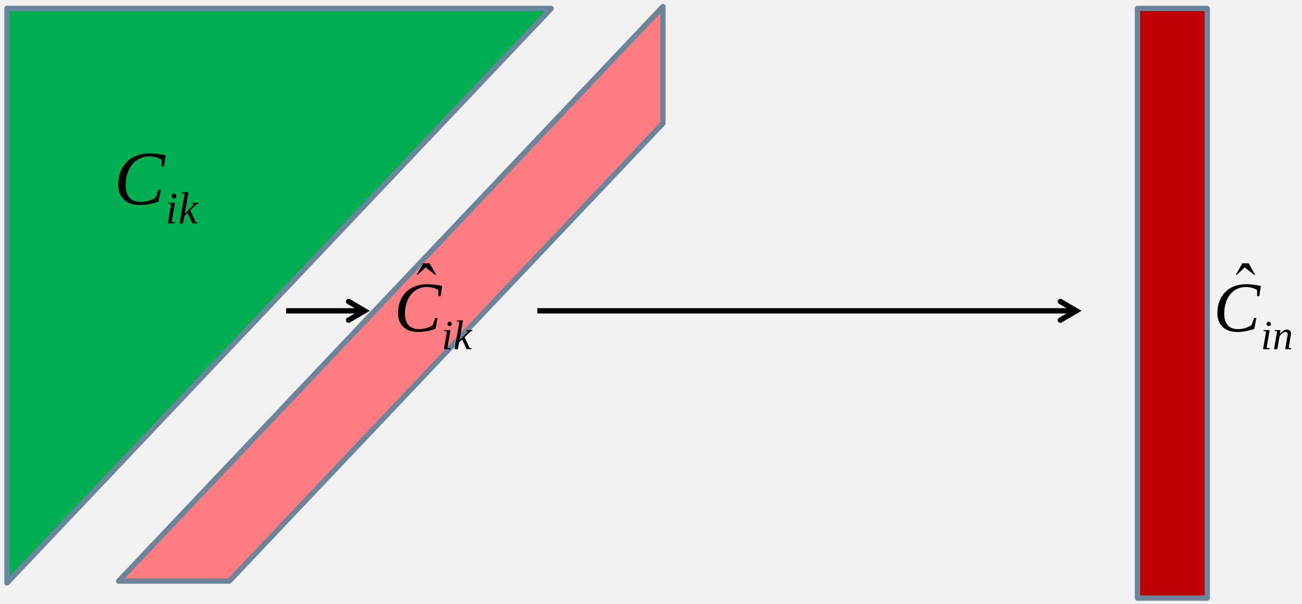
METHODOLOGICAL PERSPECTIVE
AVE ANALYSIS AND MEAN SQUARED ERROR ESTIMATES

Actual versus Expected (AvE) Analysis

Procedure in standard reserving methods (e.g. Chain-Ladder, Bornhuetter-Ferguson, Incremental-Loss-Ratio):



The estimation of the ultimate loss is done stepwise and gives also an estimate for the expected loss in the following calendar year.

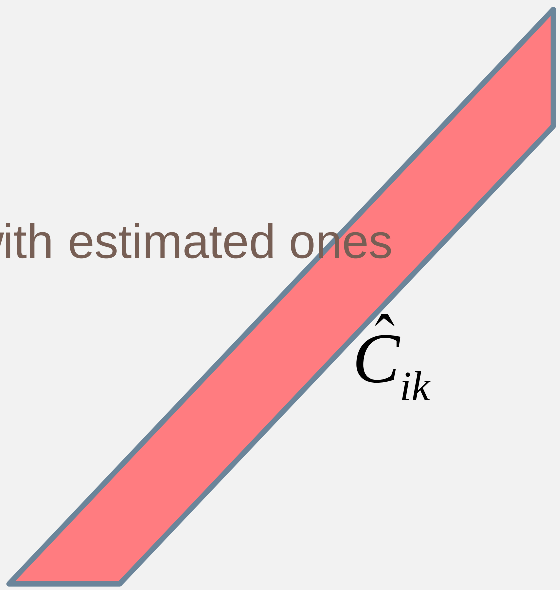
Actual versus Expected (AvE) Analysis

Monitoring in next calendar year:

The new diagonal (calendar year) is then (partially) known and can be compared with the expectation calculated in last reserve review.

Comparison of actual losses

with estimated ones



Actual versus Expected (AvE) Analysis

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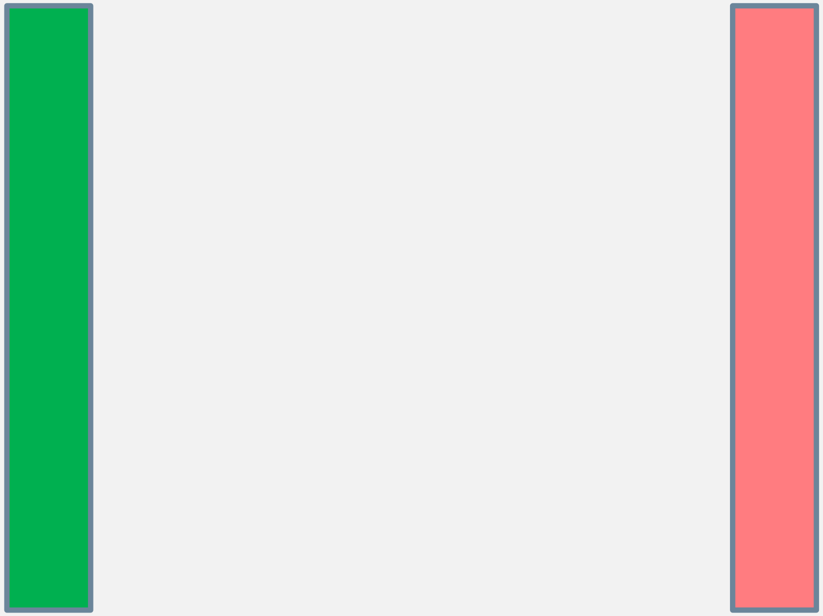
Monitoring in next calendar year:

It is sufficient to consider the increments, since any difference between Actual and Estimate is based on the difference of the increments.

Compare:



Actual: S_{ik}

Expected: $\hat{S}_{ik}$

Actual versus Expected (AvE) Analysis

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Example:

- Single segment with a development period of four years,
- Expected (losses) calculated in Reserve Review 2011,
- Actual (losses) in Financial Year 2012

Exposure Year	Actual (A)	Expected (E)	A-E	A/E
2009	100	95	5	105%
2010	120	120	0	100%
2011	115	120	-5	96%
2012	105	110	-5	95%
Total	440	445	-5	99%

Actual versus Expected (AvE) Analysis

Idea simple, but:

- Expectation for next Financial Year is required (difficult for Expected Loss Ratio Method, mix of various methods).
- Split of the expectation to financial quarters is required (depends on the statistics used) if monitoring is done on a quarterly basis.
- Carried reserves may deviate from calculation results.
- Monitoring based on paid data and reported Loss data in next Financial year.

Pattern methods are necessary, in order to break down total reserves into an expected emergence pattern.

AvE analysis and mean squared error estimates

To assess, if a deviation of actual loss emergence from expected amounts is significant, we can use the stochastic model assumptions as a basis:

One-year mean squared error by accident year

In the CL- as well as in the ILR-setting, the mean squared error of prediction

$$\text{mse}(\hat{C}_{ik}) := \text{E}((C_{ik} - \hat{C}_{ik})^2) \approx \text{Var}(C_{ik}) + \text{Var}(\hat{C}_{ik})$$

of the loss amount of the next future diagonal $i + k = n + 2$ can be easily estimated using the formulas for random error and estimation error above.

AvE analysis and mean squared error estimates

One-year mean squared error by segment

Using the independence assumptions CL2 or ILR2, respectively, the mse-estimations can be aggregated across accident years by addition:

- For the random error, additivity clearly follows from independence.
- For the one-year estimation error, additivity is a consequence of subsequent development factors (incremental loss ratios) being uncorrelated (independent).

One-year mean squared error by line or for the total portfolio

Using the model for correlated run-off triangles, the one-year mean squared errors can be aggregated across any collection of segments, in particular across a line of business or for the total portfolio.

AvE analysis and mean squared error estimates

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One-year mse-estimates can be used to benchmark the AvE deviations

- Via distribution assumptions, the quantile of AvE-deviations can be calculated. Using confidence intervals, accident years and segments that should be monitored closely can be detected automatically.
- For quaterly monitoring purposes, one can use pragmatic approaches (downscaling of mse-estimates, monitor top (e.g.) 50 deviations w.r.t. their quantiles (high and low, number depends on portfolio) or calculate quaterly mse-estimates (see slides below).

Problems with one-year mse-estimates

- Underlying model assumptions may be violated. Therefore, analysis results must be carefully interpreted.
- In particular calendar-year effects might violate the assumption of independent accident years. In this case, mse-estimates for segments or portfolios will be misleading.

AvE analysis and mean squared error estimates

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Example:

Individual Exposure Years show a deviation higher than the mse, but in total the difference is within the mse.

Exposure Year	Actual	Expected	A-E	mse
2000	32	32	0	1
2001	51	51	0	1
2002	52	52	0	0
2003	37	37	0	0
2004	98	98	0	0
2005	96	99	-3	4
2006	188	190	-2	4
2007	125	124	1	4
2008	203	205	-2	12
2009	158	165	-8	11
2010	122	116	6	8
2011	8	21	-14	3
2012	1	3	-2	2
Total	**1.170**	**1.194**	**-24**	**20**

METHODOLOGICAL PERSPECTIVE
ANSWERS TO AVE AND STRATEGIES

Answers to AvE and strategies

Actual versus Expected statistics help to detect deviations between expected and incurred loss development.

How to react on the deviation?

There is no preferable simple approach. The options listed below do not represent recommendations, but help to classify potential reactions.

Option 1: No change of Ultimate loss ratios. This means, favorable/adverse experience is not realized.

Option 2: Favorable/adverse indication is realized one for one, i.e. ultimate is reduced/increased by actual versus expected difference.

Option 3: Favorable/adverse indication is affecting forward projections and leveraged for ultimate changes.

Answers to AvE and strategies

Option 1 is

- reasonable if indication is considered as not mature enough (random noise, no trend visible or manifested enough).
- ‚natural' reaction in case of Expected-Loss-Ratio (ELR) method.

Option 2 is

- not realizing more than already manifested.
- ‚natural' reaction in case of Incremental-Loss-Ratio method and Bornhuetter-Ferguson method.

Option 3 is

- leveraging the indications.
- ‚natural' reaction in case of Chain-Ladder method.

METHODOLOGICAL PERSPECTIVE
CHECK CALCULATIONS

Check calculations

- Actual versus Expected shows the difference between actual and expected losses for the current calendar year.

- The options how to react - without doing a new full reserve study – were described in the slides above.

- Measuring the impact of the AvE on the ultimate estimate can be done by a so-called **check calculation**.

Idea: Perform the „same“ calculation as in the last reserve review, but with the new run-off triangle.

Check calculations

Perform the **„same“** calculation as in the last reserve review, but with the new run-off triangle.

Many options for „same“:

1. Use exactly the same development factors as in the last study.
2. Update the development factors with information of the new diagonal.
3. Check the impact of new diagonal for plausibility (e.g. exclusions).
4. Analyze additional impact of the new diagonal (e.g. regression).

If one is doing all four check calculations, the new study is anyway almost done.

Check calculations

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Example 3:

Exposure Year	Actual	Expected	A-E
2000	0,1	0,1	0,0
2001	0,2	0,3	-0,1
2002	0,9	1,0	-0,1
2003	0,1	0,1	0,0
2004	0,1	0,2	-0,1
2005	0,5	0,6	-0,1
2006	1,0	2,3	-1,3
2007	-3,1	3,9	-7,0
2008	1,4	1,4	0,1
2009	0,1	1,3	-1,1
2010	0,8	2,2	-1,3
2011	1,7	2,1	-0,4
2012	2,3	4,4	-2,1
Total	**6,3**	**19,9**	**-13,6**

METHODOLOGICAL PERSPECTIVE
RESIDUALS AND RESIDUAL ANALYSIS

Residuals and residual analysis

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- A **residual** measures the deviation of the “average” in multiples of the standard deviation. It is defined as standardized deviation of a random variable from its expected value.

$$\operatorname{Res}(X) := \frac{X - \operatorname{E}(X)}{\sqrt{\operatorname{Var}(X)}}$$

- More generally, we define the **conditional residual** of a random variable w.r.t. a condition by:

$$\operatorname{Res}(X \mid \Delta) := \frac{X - \operatorname{E}(X \mid \Delta)}{\sqrt{\operatorname{Var}(X \mid \Delta)}}$$

To keep notations simple, we usually omit the condition.

- Residuals are **standardized**, i.e.

$$\operatorname{E}(\operatorname{Res}(X)) = 0 \text{ and } \operatorname{Var}(\operatorname{Res}(X)) = 1$$

Residuals and residual analysis

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- **Residuals in the CL-model** are the standardized deviations of the individual development factors to the average development factor

$$\operatorname{Res}(F_{ik}) := \frac{F_{ik} - \operatorname{E}(F_{ik})}{\sqrt{\operatorname{Var}(F_{ik})}} = \frac{F_{ik} - f_k}{\sigma_k} \cdot \sqrt{C_{i,k-1}}$$

and estimated using the respective estimators.

- **Residuals in the ILR model** are the standardized deviations of the individual incremental loss ratios to the average incremental loss ratio

$$\operatorname{Res}(M_{ik}) := \frac{M_{ik} - \operatorname{E}(M_{ik})}{\sqrt{\operatorname{Var}(M_{ik})}} = \frac{M_{ik} - m_k}{s_k} \cdot \sqrt{v_i}$$

and estimated using the respective estimators.

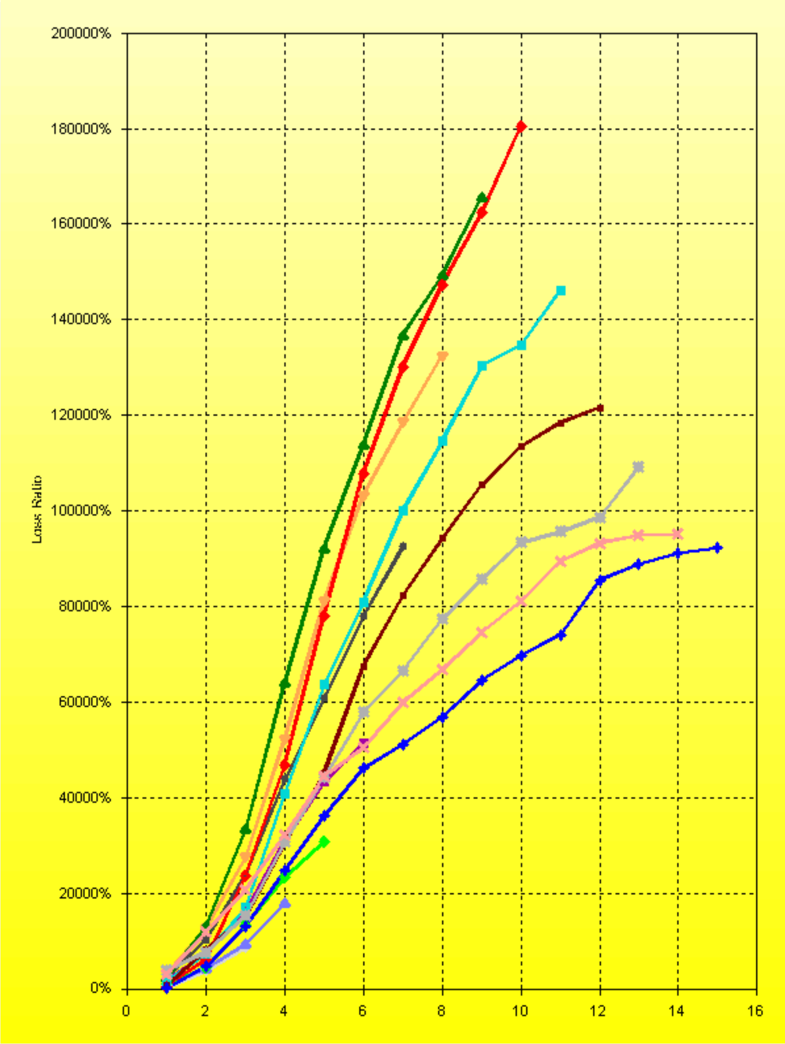
Residuals and residual analysis

Example:

The chart on the right hand side shows the reported loss ratio progression across development years for a series of accident years.

The underlying premium cycle is clearly visible.

Premium amounts have been rescaled.

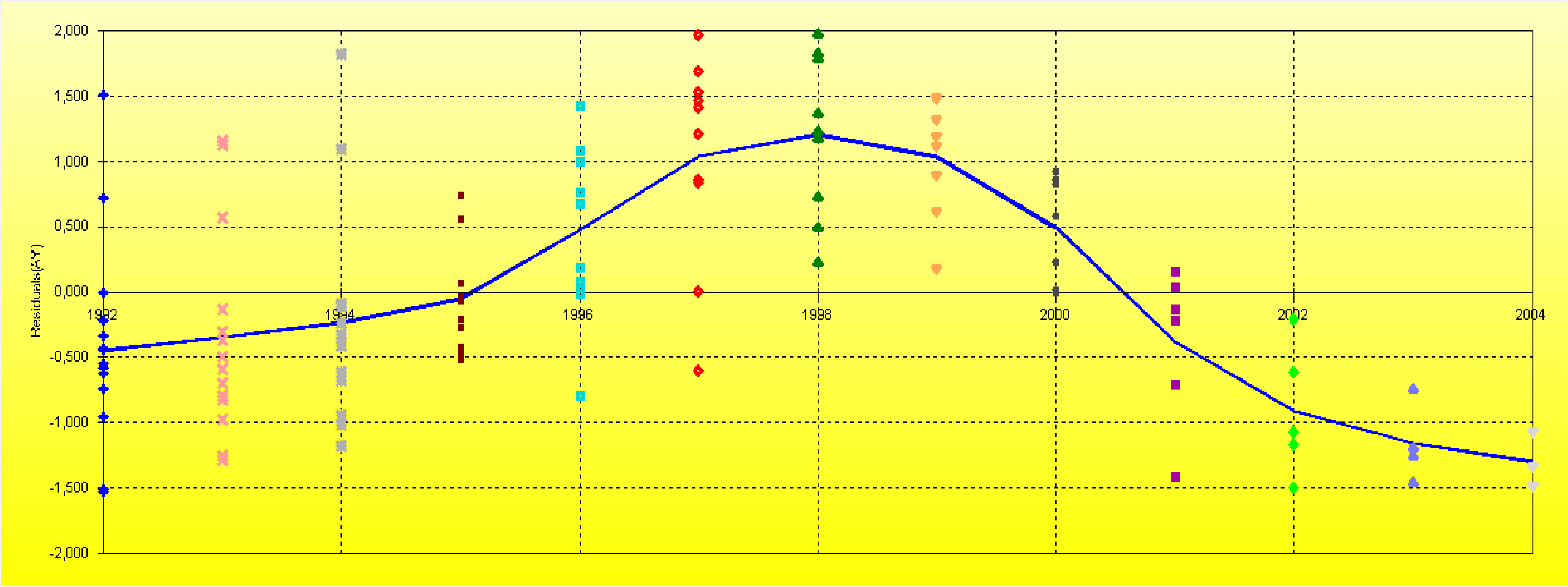


Residuals and residual analysis

Example:

The premium cycle is visible in the **accident year residual plot**, where residuals are calculated based on the ILR-model.

(Obviously, the model assumption ILR1 is not fulfilled.)



Residuals and monitoring

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Residual analysis applied to the current diagonal represents a technique to monitor loss emergence (paid or reported) of the current financial year.

Similar to the AvE analysis, historical loss emergence is “somehow” used to be compared to current loss emergence. But there are significant differences:

- Residuals do not incorporate the volume of favorable/adverse development, an important figure for management reporting.
- When actual amounts deviate from expected amounts, residuals may answer the question if the key driver is A or E, a question not directly addressed by AvE.
- Communicating details of a residual analysis is more difficult than of an AvE analysis.

Thank you very much for your attention

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