REPORT
ON THE PREPARATION OF GRADUATED TABLES OF MORTALITY RATES EXPERIENCED BY INSURED LIVES IN SINGAPORE
1988 - 1993

INTRODUCTION

I have been requested by Mrs Lim Shu Chiau, Insurance Commissioner, Singapore to prepare tables of graduated mortality rates for male and female lives insured in Singapore, under Ordinary insurances, based on experience during the five years ended 31 December 1993.

2. This task has been performed in two stages -

Stage	Task
1	Compilation of the Numbers of Lives Insured under Ordinary policies and the Numbers of Related Deaths during the period of five years 31 December 1988 to 31 December 1993.
2	Calculation of the experienced rates of mortality for successive ages and the preparation of a graduation of these rates to produce standard tables of mortality rates which progress regularly by age and represent the actual experience.

3. These tasks have been completed to provide a separate series of graduated mortality rates for male lives and for female lives in Singapore.

THE STANDARD LIFE TABLES

4. Standard life tables for male and female lives have been calculated using the graduated rates of mortality obtained by the methods described in Parts 1 and 2 of this Report, for individual ages from age 10 to age 99. The tables present the following items for each age -

Item	Definition
l_x	The number of lives surviving at exact age x.
d_x	The number of deaths in the year of age x to x + 1 among the l_x males or females who enter on that year.
p_x	The probability of a male or female aged x living a year.
q_x	The probability of a male or female aged x dying within a year.
e_x	The complete expectation of life or the average number of years lived after age x by each of a group of males or females aged exactly x.

5. The life tables which display these factors are included in the Appendix to each Part.

6. The Report is in the following Parts -

Part	Subject
	Graduated Mortality Rates for Ordinary Insurances for -
1	Male Lives Insured 1988 - 93
2	Female Lives Insured 1988 - 93

SINGAPORE - THE STANDARD LIFE TABLE

MALES

PART 1 - GRADUATED MORTALITY RATES FOR MALE LIVES INSURED - SINGAPORE - 1988 TO 1993

1.1 This investigation is based on the experience of male lives insured in Singapore, under Ordinary insurances, during the five years ended 31 December 1993.

1.2 The total data available for the purposes of the investigation for male lives is shown in Table 1.

Table 1 - Summary of the Data
Singapore - Ordinary Insurances
Males - Five Years to 31 December 1993

Item	Whole Life and Endowment Insurances		
	With Medical	Without Medical	Total
In Force - 31 December			
1988	70,000	294,529	364,529
1989	81,689	345,415	427,104
1990	84,257	364,914	449,171
1991	97,400	494,485	591,885
1992	108,204	588,669	696,873
1993	116,605	664,758	781,363
Totals	**558,155**	**2,752,770**	**3,310,925**
Deaths During -			
1989	153	341	494
1990	135	316	451
1991	152	371	523
1992	181	492	673
1993	174	455	629
Totals	**795**	**1,975**	**2,770**

1.3 A summary of the mortality experience derived from this data is presented in the next Table.

Table 2 - Experienced Rates of Mortality
Singapore - Ordinary Insurances - Duration 2+ years
Males - Various Periods

Age Group	Total Data		Singapore 1983/88 (Males Ordinary)	Ratios	
	1988/93	1983/88		(1)/(2)	(1)/(3)
	(1)	(2)	(3)	(4)	(5)
19-	.00096	.00082	.00076	1.17	1.26
24-	.00054	.00057	.00065	0.95	0.83
29-	.00049	.00060	.00059	0.82	0.83
34-	.00059	.00085	.00086	0.65	0.69
39-	.00113	.00150	.00148	0.75	0.76
44-	.00169	.00210	.00207	0.80	0.82
49-	.00259	.00377	.00397	0.69	0.65
54-	.00577	.00797	.00779	0.72	0.74
59-	.00878	.01191	.01267	0.74	0.69
64-	.01291	.01682	.01973	0.77	0.65

1.4 The following comments are made on the rates and ratios presented in Table 2.

(a) Significant reductions have occurred in the mortality rates applicable to male lives insured under Ordinary policies.

(b) The increase in the male mortality rates experienced for ages 19 to 23 years is based on an increase to 80 deaths which occurred in that age-group at Durations 2+ years. However, the reverse situation applies for Durations 0 and 1 years - Actual Deaths at ages 19 to 23, at these durations were 60, against Expected Deaths of 113.

1.5 The position regarding the availability of data at older ages and the proportions of business in force which was written without completing a medical examination are presented in Table 3.

Table 3 - The Age Distribution of the Exposed to Risk
Singapore - Ordinary Insurances - Duration 2+ Years
Males - Five Years Ended 31 December 1993

Age Group	Exposed to Risk			Proportion which is Without Medical	
	With Medical	Without Medical	Total	1988/93	1983/88
	(1)	(2)	(3)	(4)	(5)
19-	4,762	72,861	77,623	0.94	0.95
24-	16,010	219,348	235,358	0.93	0.92
29-	35,503	300,371	335,874	0.89	0.84
34-	56,348	303,068	359,416	0.84	0.77
39-	73,668	241,344	315,012	0.77	0.69
44-	60,512	132,493	193,004	0.69	0.66
49-	39,143	67,130	106,272	0.63	0.61
54-	23,531	26,924	50,454	0.53	0.48
59-	14,357	7,770	22,127	0.35	0.33
64-	6,141	2,025	8,166	0.25	0.12
Total All Ages	339,900	1,477,926	1,817,825	0.81	0.76

1.6 An increasing proportion of the exposed to risk is made up of With Medical policies as the age attained increases. There is a notable degree of consistency between the ratios shown in columns (4) and (5) of Table 3. The numbers exposed to risk decrease rapidly after age 53.

THE GRADUATED MORTALITY RATES

1.7 Graduated mortality rates have been prepared for ages 9 to 99 years, using Spencer's 21 Term Summation Formula.

1.8 The technical bases and formulae involved are explained in detail in Appendix C.

1.9 It will be noted that the procedure outlined in Appendix C involved the preparation of the graduated rates of mortality by application of Spencer's 21 Term Summation Formula to the experienced rates of mortality, after derivation of appropriate mortality rates for Ages 9 to 21 years and for ages 60 years onwards.

Ages 10 to 21 Years

1.10 Data tabulations for male lives show that, for Durations 2+ years, only 17 deaths were recorded during 1988 to 1993, in the age range 9 to 18 years. A significant accident peak occurred in the early twenties.

1.11 After consideration of a series of trials, it was decided to adopt the following rates for ages 9, 14 and 19.

Table 4 - Pivotal Mortality Rates
Males - Ages 9 to 23 Years

Age Group	Central Age	Adopted Rate
9-	11	.00024
14-	16	.00045
19-	21	.00080

Ages 56 Years Onwards

1.12 It will be observed from Table 3 that the data available from about age 60 years onwards is not sufficient to enable the calculation of reliable rates of mortality for those ages. An assumption is required therefore, as to the levels of mortality likely to be experienced at these ages.

1.13 A similar problem occurred with the preparation of the mortality tables for 1977/83 and 1983/88. The bases then adopted, was to establish from a study of the data for the period of years under consideration, an appropriate relationship between the experienced rates of mortality and the rates disclosed by a standard mortality table.

1.14 Examination of the ratios in column (5) of Table 2 and of the limited data available at ages 59 and over indicate that a ratio of 75% of the rates in the Singapore 1983/88 Males Mortality Table from Age 60 onwards would be appropriate.

1.15 The details of the graduation by Spencer's Method are set out in Appendix C. A summary of the results is in Table 5.

Table 5 - Comparison of Actual and Expected Deaths
Spencer's 21 Term Formula - Total Data
Singapore - Ordinary Insurances - 2+ Years
Males - Five Years ended 31 December 1993

Age Group	Deaths		Difference = (1) - (2)		
	Actual	Expected	+	-	Sum
	(1)	(2)	(3)	(4)	(5)
9-	9	9	-	-	-
14-	10	36	16	26	-25*
19-	80	63	-	-	-9
24-	138	154	-	16	-25
29-	185	175	10	-	-15
34-	250	270	-	21	-36*
39-	442	415	27	-	-9
44-	431	419	12	-	3
49-	378	418	-	40	-37
54-	429	412	17	-	-20
59-	322	366	-	44	-63*
64-	186	229	-	43	-106
Totals	2,860	2,966	82	190	-108

* *Rounding*

1.17 Examination of Table 5 shows that -

(a) The graduation has exercised a significant smoothing influence over the variations in the experienced rates of mortality for successive age-groups.

(b) The graduation has resulted in a table of mortality rates which, whilst adopting a conservative basis for older ages, provides a satisfactory standard for use and for comparisons in future reports.

1.25 Rates of mortality for each age are show in Appendix A, following this Part.

APPENDIX A

SINGAPORE - THE GRADUATED MORTALITY RATES AND COMPARISONS OF ACTUAL AND EXPECTED DEATHS MALES

SINGAPORE

MORTALITY EXPERIENCE - 1988 - 1993 - MALES

WHOLE LIFE & ENDOWMENT INSURANCES - ORDINARY INSURANCES

GRADUATED MORTALITY RATES - SPENCERS 21 TERM SUMMATION FORMULA

AGE	GRADUATED RATE
9	0.00021
10	0.00022
11	0.00023
12	0.00025
13	0.00029
14	0.00033
15	0.00037
16	0.00042
17	0.00048
18	0.00054
19	0.00061
20	0.00068
21	0.00076
22	0.00082
23	0.00080
24	0.00077
25	0.00072
26	0.00065
27	0.00058
28	0.00052
29	0.00048
30	0.00046
31	0.00046
32	0.00046
33	0.00048
34	0.00051
35	0.00055
36	0.00061
37	0.00068
38	0.00076
39	0.00085
40	0.00094
41	0.00103
42	0.00113
43	0.00123
44	0.00135
45	0.00148
46	0.00162
47	0.00177
48	0.00195
49	0.00216
50	0.00242
51	0.00273
52	0.00311
53	0.00357
54	0.00411
55	0.00473
56	0.00545
57	0.00625
58	0.00711
59	0.00802
60	0.00896

61	0.00992
62	0.01093
63	0.01197
64	0.01310
65	0.01437
66	0.01582
67	0.01748
68	0.01939
69	0.02153
70	0.02389
71	0.02646
72	0.02920
73	0.03213
74	0.03527
75	0.03864
76	0.04225
77	0.04615
78	0.05037
79	0.05492
80	0.05983
81	0.06513
82	0.07082
83	0.07694
84	0.08348
85	0.09046
86	0.09786
87	0.10563
88	0.11370
89	0.12200
90	0.13050
91	0.13928
92	0.14859
93	0.15885
94	0.17059
95	0.18436
96	0.20059
97	0.21953
98	0.24120
99	0.26543

SINGAPORE

MORTALITY EXPERIENCE - 1988 - 1993 - MALES

WHOLE LIFE & ENDOWMENT INSURANCES - ORDINARY INSURANCES

GRADUATED MORTALITY RATES - SPENCERS 21 TERM SUMMATION FORMULA

AGE	GRADUATED RATE	ACTUAL RATE	ACTUAL DEATHS	EXPECTED DEATHS	ACTUAL - EXPECTED	SUM
GRAD.MORT RATES STORED IN FILE			SINGM-93			
9	0.00021					
10	0.00022					
11	0.00023					
12	0.00025					
13	0.00029					
9	0.00023	0.00024	9	9	0	0
14	0.00033					
15	0.00037					
16	0.00042					
17	0.00048					
18	0.00054					
14	0.00044	0.00012	10	36	-26	-25
19	0.00061					
20	0.00068					
21	0.00076					
22	0.00082					
23	0.00080					
19	0.00076	0.00096	80	63	16	-9
24	0.00077					
25	0.00072					
26	0.00065					
27	0.00058					
28	0.00052					
24	0.00061	0.00054	138	154	-16	-25
29	0.00048					
30	0.00046					
31	0.00046					
32	0.00046					
33	0.00048					
29	0.00047	0.00049	185	175	9	-15
34	0.00051					
35	0.00055					
36	0.00061					
37	0.00068					
38	0.00076					
34	0.00064	0.00059	250	270	-21	-36
39	0.00085					
40	0.00094					

41	0.00103					
42	0.00113					
43	0.00123					
39	0.00106	0.00113	442	415	27	-9
44	0.00135					
45	0.00148					
46	0.00162					
47	0.00177					
48	0.00195					
44	0.00164	0.00169	431	419	12	3
49	0.00216					
50	0.00242					
51	0.00273					
52	0.00311					
53	0.00357					
49	0.00287	0.00259	378	418	-40	-37
54	0.00411					
55	0.00473					
56	0.00545					
57	0.00625					
58	0.00711					
54	0.00554	0.00577	429	412	17	-20
59	0.00802					
60	0.00896					
61	0.00992					
62	0.01093					
63	0.01197					
59	0.00996	0.00878	322	366	-44	-63
64	0.01310					
65	0.01437					
66	0.01582					
67	0.01748					
68	0.01939					
64	0.01585	0.01291	186	229	-43	-106
69	0.02153					
70	0.02389					
71	0.02646					
72	0.02920					
73	0.03213					
69	0.02603	0.02670	136	133	3	-103
74	0.03527					
75	0.03864					
76	0.04225					
77	0.04615					
78	0.05037					
74	0.04133	0.03883	70	75	-5	-107
79	0.05492					
80	0.05983					

81	0.06513					
82	0.07082					
83	0.07694					
79	0.06287	0.06990	46	41	5	-103
84	0.08348					
85	0.09046					
86	0.09786					
87	0.10563					
88	0.11370					
84	0.11036					
89	0.12200					
90	0.13050					
91	0.13928					
92	0.14859					
93	0.15885					
89	0.00000					
94	0.17059					
95	0.18436					
96	0.20059					
97	0.21953					
98	0.24120					
94	0.00000					
99	0.26543					
99	0.00000					

NOTES: GRADUATED RATES FROM AGE 9 TO 99 STORED IN FILE SINGM-93
RATES FROM AGE 60 ARE BASED ON 75 % OF FILE NO. 18

SINGAPORE - THE STANDARD LIFE TABLE

FEMALES

AGE	LIVING	DEATHS	PROBABILITIES		EXPECTATION OF LIFE
			px	qx	
58	938315	6671	.99289	.00711	24.13
59	931644	7472	.99198	.00802	23.30
60	924172	8281	.99104	.00896	22.48
61	915891	9086	.99008	.00992	21.67
62	906805	9911	.98907	.01093	20.88
63	896894	10736	.98803	.01197	20.10
64	886158	11609	.98690	.01310	19.33
65	874549	12567	.98563	.01437	18.57
66	861982	13637	.98418	.01582	17.82
67	848345	14829	.98252	.01748	17.09
68	833516	16162	.98061	.01939	16.38
69	817354	17598	.97847	.02153	15.68
70	799756	19106	.97611	.02389	15.00
71	780650	20656	.97354	.02646	14.34
72	759994	22192	.97080	.02920	13.70
73	737802	23706	.96787	.03213	13.08
74	714096	25186	.96473	.03527	12.48
75	688910	26619	.96136	.03864	11.90
76	662291	27982	.95775	.04225	11.34
77	634309	29273	.95385	.04615	10.79
78	605036	30476	.94963	.05037	10.26
79	574560	31555	.94508	.05492	9.75
80	543005	32488	.94017	.05983	9.26
81	510517	33250	.93487	.06513	8.78
82	477267	33800	.92918	.07082	8.32
83	443467	34120	.92306	.07694	7.88
84	409347	34172	.91652	.08348	7.45
85	375175	33938	.90954	.09046	7.04
86	341237	33393	.90214	.09786	6.65
87	307844	32518	.89437	.10563	6.26
88	275326	31305	.88630	.11370	5.89
89	244021	29771	.87800	12200	5.52
90	214250	27960	.86950	.13050	5.16
91	186290	25946	.86072	.13928	4.79
92	160344	23826	.85141	.14859	4.42
93	136518	21686	.84115	.15885	4.03
94	114832	19589	.82941	.17059	3.62
95	95243	17559	.81564	.18436	3.18
96	77684	15583	.79941	.20059	2.70
97	62101	13633	.78047	.21953	2.16
98	48468	11690	.75880	.24120	1.52
99	36778	9762	.73457	.26543	0.85

SINGAPORE MALES

SINGAPORE1988/93(M)

AGE	LIVING	DEATHS	PROBABILITIES px	PROBABILITIES qx	EXPECTATION OF LIFE
9	1000000	210	.99979	.00021	70.88
10	999790	220	.99978	.00022	69.90
11	999570	230	.99977	.00023	68.91
12	999340	250	.99975	.00025	67.93
13	999090	290	.99971	.00029	66.95
14	998800	330	.99967	.00033	65.97
15	998470	369	.99963	.00037	64.99
16	998101	419	.99958	.00042	64.01
17	997682	479	.99952	.00048	63.04
18	997203	538	.99946	.00054	62.07
19	996665	608	.99939	.00061	61.10
20	996057	677	.99932	.00068	60.14
21	995380	756	.99924	.00076	59.18
22	994624	816	.99918	.00082	58.22
23	993808	795	.99920	.00080	57.27
24	993013	765	.99923	.00077	56.31
25	992248	714	.99928	.00072	55.36
26	991534	644	.99935	.00065	54.40
27	990890	575	.99942	.00058	53.43
28	990315	515	.99948	.00052	52.46
29	989800	475	.99952	.00048	51.49
30	989325	455	.99954	.00046	50.51
31	988870	455	.99954	.00046	49.53
32	988415	455	.99954	.00046	48.56
33	987960	474	.99952	.00048	47.58
34	987486	504	.99949	.00051	46.60
35	986982	543	.99945	.00055	45.62
36	986439	602	.99939	.00061	44.65
37	985837	670	.99932	.00068	43.68
38	985167	749	.99924	.00076	42.70
39	984418	837	.99915	.00085	41.74
40	983581	925	.99906	.00094	40.77
41	982656	1012	.99897	.00103	39.81
42	981644	1109	.99887	.00113	38.85
43	980535	1206	.99877	.00123	37.89
44	979329	1322	.99865	.00135	36.94
45	978007	1447	.99852	.00148	35.98
46	976560	1582	.99838	.00162	35.04
47	974978	1726	.99823	.00177	34.09
48	973252	1898	.99805	.00195	33.15
49	971354	2098	.99784	.00216	32.21
50	969256	2346	.99758	.00242	31.28
51	966910	2640	.99727	.00273	30.35
52	964270	2999	.99689	.00311	29.43
53	961271	3432	.99643	.00357	28.52
54	957839	3937	.99589	.00411	27.62
55	953902	4512	.99527	.00473	26.73
56	949390	5174	.99455	.00545	25.85
57	944216	5901	.99375	.00625	24.98

Table 7 - Experienced Rates of Mortality
Singapore - Ordinary Insurances - Duration 2+ Years
Females

Age Group	Total Data		Singapore 1983/88 (Females)	Ratios	
	1988/93	1983/88		(1)/(2)	(1)/(3)
	(1)	(2)	(3)	(4)	(5)
19-	.00038	.00021	.00026	1.81	1.46
24-	.00024	.00020	.00029	1.20	0.83
29-	.00030	.00045	.00046	0.67	0.65
34-	.00042	.00074	.00073	0.57	0.58
39-	.00066	.00074	.00086	0.89	0.77
44-	.00117	.00124	.00134	0.94	0.87
49-	.00180	.00227	.00252	0.79	0.71
54-	.00227	.00340	.00480	0.67	0.47
59-	.00425	.00692	.00831	0.61	0.51
64-	.00719	.01134	.01311	0.63	0.55

2.4 Column (5) of Table 7 shows some deterioration in the experienced rates of mortality for female insured lives at the younger ages, followed by a generally reducing trend for the subsequent ages attained.

2.5 The division of the numbers exposed to risk between policies written with and without a medical examination is show in Table 8.

Table 8 - The Age Distribution of the Exposed to Risk
Singapore - Ordinary Insurances - Duration 2+ Years
Females - Five Years Ended 31 December 1993

Age Group	Exposed to Risk			Proportion which is Without Medical
	With Medical	Without Medical	Total	
	(1)	(2)	(3)	(4)
19-	3,582	57,457	61,039	0.94
24-	13,763	194,829	208,592	0.93
29-	23,424	244,140	267,564	0.91
34-	25,659	222,457	248,116	0.90
39-	24,172	159,293	183,465	0.87
44-	18,504	86,428	104,932	0.82
49-	12,534	38,689	51,233	0.76
54-	10,910	13,246	24,156	0.55
59-	6,733	3,891	10,624	0.37
64-	3,337	1,498	4,835	0.31
Total (All ages)	148,725	1,104,382	1,253,107	0.88

2.6 It is a notable feature of the above table that a very high proportion of the exposed to risk is made up of policies issued without a medical examination for age groups up to ages 44 to 48 years. The proportion in this category decreases progressively and quite quickly for the higher age-groups.

THE GRADUATED MORTALITY RATES

2.7 Graduated mortality rates for female lives have been prepared for ages 10 to 99 years, using Spencer's 21 Term Summation Formula.

2.8 The technical bases and formulae involved are explained in detail in Appendix C, similar procedures having been adopted for male and female lives.

2.9 These procedures involve the preparation of the graduated rates of mortality in two sectors, as described in the next paragraphs.

Ages 10 to 21 Years

2.10 Pivotal mortality rates were determined from the experience for female insured lives during 1988 to 1993, as follows

Table 9 - Pivotal Mortality Rates
Females - Ages 9 to 23 years - 1988 to 1993

Age Groups	Central Age (Years)	Adopted Rate
9-	11	.00004
14-	16	.00015
19-	21	.00039

Ages 54 Years Onwards

2.10 As the data available for ages 60 and later ages proved to be not sufficient to enable the calculation of reliable rates of mortality for these ages, it was necessary to make a judgement as to the mortality rates which might be experienced in future at these ages.

2.11 The formula decided upon for the older ages for female lives is -

75% of Singapore 1983/88 (F-ORD) Table from age 60 years subject to special determination for ages 90 to 99 years.

2.12 The graduation of the mortality rates for female lives is set out in Appendix B. A summary of the results is in Table 10.

Table 10 - Comparison of Actual and Expected Deaths
Spencer's 21 Term Formula - Total Data
Singapore - Ordinary Insurances - 2+ Years
Females - Five Years ended 31 December 1993

Age Group	Deaths		Difference = (1) - (2)		
	Actual	Expected	+	-	Sum
	(1)	(2)	(3)	(4)	(5)
9-	1	2	-	1	-1
14-	9	9	-	-	-1
19-	25	21	4	7	3
24-	55	62	-	-	-5*
29-	88	87	1	-	-4
34-	117	116	1	-	-3
39-	138	140	-	2	-5
44-	145	143	2	-	-3
49-	115	113	2	-	-1
54-	80	77	3	-	1*
59-	74	98	-	24	-23
64-	59	88	-	29	-53
Totals	**906**	**956**	**13**	**63**	**-50**

* *Rounding*

2.13 Examination of Table 10 shows that -

(a) The deviations between the numbers of actual and expected deaths at each age-group are small.

(b) For ages 60 onwards, the choice of expected mortality rates equal to 75% of the rates in the Singapore Females 1983/88 Table is considered to represent a conservative view of the future expected mortality experience.

2.14 Rates of mortality for each age are shown in Appendix B, following this Part.

S W CAFFIN

23 November 1995

APPENDIX B

SINGAPORE - THE GRADUATED MORTALITY RATES AND COMPARISONS OF ACTUAL AND EXPECTED DEATHS FEMALES

SINGAPORE

MORTALITY EXPERIENCE - 1988 - 1993 - FEMALES

WHOLE LIFE & ENDOWMENT INSURANCES - ORDINARY INSURANCES

GRADUATED MORTALITY RATES - SPENCERS 21 TERM SUMMATION FORMULA

AGE	GRADUATED RATE
9	0.00002
10	0.00003
11	0.00004
12	0.00005
13	0.00006
14	0.00008
15	0.00010
16	0.00013
17	0.00017
18	0.00021
19	0.00025
20	0.00030
21	0.00036
22	0.00037
23	0.00031
24	0.00031
25	0.00030
26	0.00028
27	0.00027
28	0.00027
29	0.00027
30	0.00027
31	0.00029
32	0.00031
33	0.00033
34	0.00035
35	0.00038
36	0.00041
37	0.00044
38	0.00048
39	0.00052
40	0.00058
41	0.00064
42	0.00072
43	0.00081
44	0.00091
45	0.00102
46	0.00114
47	0.00127
48	0.00141
49	0.00155
50	0.00169
51	0.00180
52	0.00187
53	0.00188
54	0.00187
55	0.00188
56	0.00201
57	0.00231
58	0.00284
59	0.00358
60	0.00450

61	0.00552
62	0.00660
63	0.00764
64	0.00865
65	0.00964
66	0.01065
67	0.01175
68	0.01297
69	0.01435
70	0.01587
71	0.01752
72	0.01928
73	0.02116
74	0.02318
75	0.02534
76	0.02767
77	0.03018
78	0.03292
79	0.03590
80	0.03913
81	0.04265
82	0.04645
83	0.05051
84	0.05479
85	0.05923
86	0.06371
87	0.06808
88	0.07221
89	0.07604
90	0.07972
91	0.08368
92	0.08874
93	0.09606
94	0.10701
95	0.12298
96	0.14516
97	0.17454
98	0.21219
99	0.25959

SINGAPORE

MORTALITY EXPERIENCE - 1988 - 1993 - FEMALES

WHOLE LIFE & ENDOWMENT INSURANCES - ORDINARY INSURANCES

GRADUATED MORTALITY RATES - SPENCERS 21 TERM SUMMATION FORMULA

AGE	GRADUATED RATE	ACTUAL RATE	ACTUAL DEATHS	EXPECTED DEATHS	ACTUAL - EXPECTED	SUM
GRAD.MORT RATES STORED IN FILE			SINGF.93			
9	0.00002					
10	0.00003					
11	0.00004					
12	0.00005					
13	0.00006					
9	0.00005	0.00003	1	2	-1	-1
14	0.00008					
15	0.00010					
16	0.00013					
17	0.00017					
18	0.00021					
14	0.00014	0.00014	9	9	-0	-1
19	0.00025					
20	0.00030					
21	0.00036					
22	0.00037					
23	0.00031					
19	0.00032	0.00038	25	21	4	3
24	0.00031					
25	0.00030					
26	0.00028					
27	0.00027					
28	0.00027					
24	0.00027	0.00024	55	62	-8	-5
29	0.00027					
30	0.00027					
31	0.00029					
32	0.00031					
33	0.00033					
29	0.00029	0.00030	88	87	1	-4
34	0.00035					
35	0.00038					
36	0.00041					
37	0.00044					
38	0.00048					
34	0.00042	0.00042	117	116	1	-3
39	0.00052					
40	0.00058					

41	0.00064					
42	0.00072					
43	0.00081					
39	0.00067	0.00066	138	140	-2	-5
44	0.00091					
45	0.00102					
46	0.00114					
47	0.00127					
48	0.00141					
44	0.00115	0.00117	145	143	2	-3
49	0.00155					
50	0.00169					
51	0.00180					
52	0.00187					
53	0.00188					
49	0.00177	0.00180	115	113	2	-1
54	0.00187					
55	0.00188					
56	0.00201					
57	0.00231					
58	0.00284					
54	0.00220	0.00227	80	77	3	1
59	0.00358					
60	0.00450					
61	0.00552					
62	0.00660					
63	0.00764					
59	0.00564	0.00425	74	98	-24	-23
64	0.00865					
65	0.00964					
66	0.01065					
67	0.01175					
68	0.01297					
64	0.01069	0.00719	59	88	-29	-52
69	0.01435					
70	0.01587					
71	0.01752					
72	0.01928					
73	0.02116					
69	0.01732	0.01638	47	50	-3	-55
74	0.02318					
75	0.02534					
76	0.02767					
77	0.03018					
78	0.03292					
74	0.02673	0.01495	15	27	-12	-66
79	0.03590					
80	0.03913					

81	0.04265					
82	0.04645					
83	0.05051					
79	0.04258	0.02162	6	12	-6	-72
84	0.05479					
85	0.05923					
86	0.06371					
87	0.06808					
88	0.07221					
84	0.07103					
89	0.07604					
90	0.07972					
91	0.08368					
92	0.08874					
93	0.09606					
89	0.00000					
94	0.10701					
95	0.12298					
96	0.14516					
97	0.17454					
98	0.21219					
94	0.00000					
99	0.25959					
99	0.00000					

SINGAPORE FEMALES

SINGAPORE88/93(F-O

AGE	LIVING	DEATHS	PROBABILITIES px	PROBABILITIES qx	EXPECTATION OF LIFE
9	1000000	20	.99998	.00002	75.78
10	999980	30	.99997	.00003	74.78
11	999950	40	.99996	.00004	73.78
12	999910	50	.99995	.00005	72.78
13	999860	60	.99994	.00006	71.79
14	999800	80	.99992	.00008	70.79
15	999720	100	.99990	.00010	69.80
16	999620	130	.99987	.00013	68.80
17	999490	170	.99983	.00017	67.81
18	999320	210	.99979	.00021	66.82
19	999110	250	.99975	.00025	65.84
20	998860	300	.99970	.00030	64.85
21	998560	359	.99964	.00036	63.87
22	998201	369	.99963	.00037	62.89
23	997832	309	.99969	.00031	61.92
24	997523	309	.99969	.00031	60.94
25	997214	299	.99970	.00030	59.95
26	996915	279	.99972	.00028	58.97
27	996636	269	.99973	.00027	57.99
28	996367	269	.99973	.00027	57.00
29	996098	269	.99973	.00027	56.02
30	995829	269	.99973	.00027	55.03
31	995560	289	.99971	.00029	54.05
32	995271	309	.99969	.00031	53.06
33	994962	328	.99967	.00033	52.08
34	994634	348	.99965	.00035	51.10
35	994286	378	.99962	.00038	50.11
36	993908	408	.99959	.00041	49.13
37	993500	437	.99956	.00044	48.15
38	993063	477	.99952	.00048	47.17
39	992586	516	.99948	.00052	46.20
40	992070	575	.99942	.00058	45.22
41	991495	635	.99936	.00064	44.24
42	990860	713	.99928	.00072	43.27
43	990147	802	.99919	.00081	42.30
44	989345	900	.99909	.00091	41.34
45	988445	1008	.99898	.00102	40.37
46	987437	1126	.99886	.00114	39.41
47	986311	1253	.99873	.00127	38.46
48	985058	1389	.99859	.00141	37.50
49	983669	1525	.99845	.00155	36.56
50	982144	1660	.99831	.00169	35.61
51	980484	1765	.99820	.00180	34.67
52	978719	1830	.99813	.00187	33.73
53	976889	1837	.99812	.00188	32.79
54	975052	1823	.99813	.00187	31.85
55	973229	1830	.99812	.00188	30.91
56	971399	1953	.99799	.00201	29.96
57	969446	2239	.99769	.00231	29.02

AGE	LIVING	DEATHS	PROBABILITIES		EXPECTATION OF LIFE
			px	qx	
58	967207	2747	.99716	.00284	28.09
59	964460	3453	.99642	.00358	27.16
60	961007	4325	.99550	.00450	26.26
61	956682	5281	.99448	.00552	25.37
62	951401	6279	.99340	.00660	24.50
63	945122	7221	.99236	.00764	23.66
64	937901	8113	.99135	.00865	22.83
65	929788	8963	.99036	.00964	22.02
66	920825	9807	.98935	.01065	21.23
67	911018	10704	.98825	.01175	20.44
68	900314	11677	.98703	.01297	19.67
69	888637	12752	.98565	.01435	18.92
70	875885	13900	.98413	.01587	18.18
71	861985	15102	.98248	.01752	17.45
72	846883	16328	.98072	.01928	16.74
73	830555	17575	.97884	.02116	16.05
74	812980	18845	.97682	.02318	15.38
75	794135	20123	.97466	.02534	14.72
76	774012	21417	.97233	.02767	14.07
77	752595	22713	.96982	.03018	13.44
78	729882	24028	.96708	.03292	12.83
79	705854	25340	.96410	.03590	12.23
80	680514	26629	.96087	.03913	11.64
81	653885	27888	.95735	.04265	11.07
82	625997	29078	.95355	.04645	10.52
83	596919	30150	.94949	.05051	9.98
84	566769	31053	.94521	.05479	9.46
85	535716	31730	.94077	.05923	8.95
86	503986	32109	.93629	.06371	8.45
87	471877	32125	.93192	.06808	7.96
88	439752	31754	.92779	.07221	7.46
89	407998	31024	.92396	.07604	6.97
90	376974	30052	.92028	.07972	6.46
91	346922	29030	.91632	.08368	5.94
92	317892	28210	.91126	.08874	5.39
93	289682	27827	.90394	.09606	4.82
94	261855	28021	.89299	.10701	4.23
95	233834	28757	.87702	.12298	3.61
96	205077	29769	.85484	.14516	2.97
97	175308	30598	.82546	.17454	2.30
98	144710	30706	.78781	.21219	1.58
99	114004	29594	.74041	.25959	0.87

APPENDIX C

COMMITTEE FOR MORTALITY STUDIES OF ASSURED LIVES

THE METHODS ADOPTED FOR THE PREPARATION OF GRADUATED MORTALITY RATES AND A STANDARD LIFE TABLE

THE GRADUATION METHOD ADOPTED

1. The method of graduation adopted is the 21 term formula adopted by the late John Spencer F.I.A.

2. This formula was used in the course of preparing the Singapore 1977/83 and 1983/88 (Ordinary) Tables and is considered suitable for the present graduation, for the following reasons -

(a) The volume of data available from the tabulations of Ordinary insurance in force in Singapore during 1988 through to 1993 is sufficient to permit the application of graduation procedures involving the use of data for individual ages, over the most important age range of 20 to 60 or 65 years.

(b) The experienced rates of mortality have been found to progress with satisfactory regularity over the main age range.

(c) The procedure adopted produces graduated rates of mortality which adhere closely to the original data. They cope very well with the important features of the ungraduated mortality rates. In particular, the accident lump which is evident about age 20 years and the requirement to blend the graduated rates smoothly with the rates adopted for the young and older ages where there is inadequate data.

(d) Spencer's 21 Term formula is suitable for computer operation.

SPENCER'S 21 TERM SUMMATION FORMULA

3. The rates of mortality for numerous mortality tables have been prepared using the graduation formula known as Spencer's 21 Term Summation Formula. It is considered to be one of the best summation formulae for graduation purposes.

4. The formula, in short form, is as follows -

$$U_x = \frac{[5]^2[7]}{350} \left([1] + [3] + [5] - [7] \right) {}^{1}U x$$

Where: U_x = the graduated value at age x

[n] ${}^{1}U x$ = the sum of n ungraduated values, the central value of which is ${}^{1}U x$.

An expanded version of the formula is in Attachment No. 1.

5. For the preparation of the graduated rates of mortality dealt with in this Report, the graduation was performed in two sections, as described in the following paragraphs.

Graduated Rates of Mortality for Ages 10 to 21 Years

6. The data which is available regarding the mortality experience by lives insured who are under 22 years of age is very limited. Data is not available for ages 0 to 8. Only very few policies issued at these ages are included in the annual returns received.

7. In the absence of other evidence, values of the average rates of mortality for the central ages of the age groups 9 ½ to 13 ½, 14 ½ to 18 ½ and 19 ½ to 23 ½ years were adopted (as described in the Report) as follows -

Pivotal Mortality Rates

Central Age	Males	Females
11 ½	.00024	.00004
16 ½	.00012	.00015
21 ½	.00090	.00039

8. An interpolation based on these rates was prepared to obtain rates of mortality for male and female lives, for ages 12 to 21 years. The formulae used are shown in Attachment No. 1.

Graduated Rates for Mortality for Ages 22 to 99 Years - Spencer's Formula

9. Examination of the volume of the data available indicated that rates of mortality for individual ages could be used for graduation purposes in the range from age 21 ½ to about age 57 ½.

Ages 10 ½ to 20 ½ Years

10. In order to obtain the greatest benefit from using Spencer's 21 Term Summation formula, it was decided to calculate rates of mortality for individual ages 10 ½ to 20 ½ years before applying the formula. The methods used for this task are described in Attachment 1.

Ages 56 ½ to 99 ½ Years

11. As a result of the investigations reported in Parts 1 and 2 of this Report, rates of mortality for individual ages 56 to 99 were assumed to equal -

For Males	-	75% of Singapore 1983/88 (M-ORD)
For Females		75% of Singapore 1983/88 (F-ORD)

12. As a result of these preliminary arrangements, it was possible to apply Spencer's 21 Term Summation Formula to obtain graduated rates of mortality for ages 11 ½ through to 99 ½. Graduated rates for ages 22 to 99 were then calculated using the interpolation formula described in Attachment No. 1.

15. The graduations are based on the experience of policies which had been 2 or more years in force. This basis was adopted to allow for the initial selection which might be exercised by companies contributing to the data, through the medical examination made prior to issue of a policy or by means of the personal statement made by the applicant for insurance.

ATTACHMENT NO. 1

THE TECHNICAL ASPECTS

SPENCER'S 21 TERM SUMMATION FORMULA

$$U_{\frac{1}{x}} = \frac{[5]^2[7]}{350}([1]+[3]+[5]-[7])U_x$$

$$= [60U_o + 57(U_{+1} + U_{-1}) + 47(U_{+2} + U_{-2}) + 33(U_{+3} + U_{-3})$$

$$+ 18(U_{+4} + U_{-4}) + 6(U_{+5} + U_{-5}) - 2(U_{+6} + U_{-6})$$

$$- 5(U_{+7} + U_{-7}) - 5(U_{+8} + U_{-8}) - 3(U_{+9} + U_{-9})$$

$$- (U_{+10} + U_{-10}) \;]/350$$

STIRLING'S FORMULA

$$\mu_x = U_o + \frac{x}{2}(\Delta U_o + \Delta U_{-1}) + \frac{x^2}{2}\Delta^2 U_{-1} + \frac{x(x^2-1)}{12}(\Delta^3 U_{-1} + \Delta^3 U_{-2})$$

Which, for $x = ½$, becomes -

$$\mu_5 = .75U_o + .4375U_1 + .03125U_2 - .1875U_{-1} - .03125U_{-2}$$

GRADUATION - AGES 10 TO 21

1. Ages 17 to 21 Years

$$U_{1+t} = U_1 + t\,\Delta U_o + \frac{t(t+1)}{2}\Delta^2 U_o$$

$$= \frac{t(t-1)}{2}U_o + (1-t^2)U_1 + \frac{t(t+1)}{2}U_2$$

Age	t	$t(t-1)/2$	$(1-t^2)$	$t(t+1)/2$
17	.1	-.045	.990	.055
18	.3	-.105	.910	.195
19	.5	-.125	.750	.375
20	.7	-.105	.510	.595
21	.9	-.045	.190	.855

Age	t			
17½	.2	-.010	.960	.120
18½	.4	-.120	.840	.280
19½	.6	-.120	.640	.480
20½	.8	-.080	.360	.720
21½	1.0	-	-	-

2. Ages 12 to 16 Years

$$U_t = U_o + t\,\Delta U_o + \frac{t(t-1)}{2}\Delta^2 U_o$$

$$= \frac{(t-1)(t-2)}{2}U_o + t(2-t)U_1 + \frac{t(t-1)}{2}U_2$$

Age	t	$(t-1)(t-2)/2$	$t(2-t)$	$t(t-1)/2$
12	.1	.853	.190	-.045
13	.3	.595	.510	-.105
14	.5	.375	.750	-.125
15	.7	.195	.910	-.105
16	.9	.055	.990	-.045

CALCULATION FOR HIGH AGES
SPENCER

1. It is assumed that -

   (a) Notional rates of mortality for ages 100 to 111 can be calculated on the basis that the rate of increase from one age to the next is -

   $$S1 = (q_{99} / q_{94})^{1/5}$$

   (b) The Spencer formula is to reproduce the known value for q_{99}.

2. The application of Spencer's formula to (b) is -

   $$q_{99} = (60q_{99} + K + K1)/350$$

Where $K = 57q_{98} + 47q_{97} + 33q_{96} + 18q_{95} + 6q_{94} - 2q_{93} - 5q_{92} - 5q_{91} - q_{90}$
and these values are known.

$$K1 = q_{100}[57 + 47S1 + 33S1^2 + 18S1^3 + 6S1^4 - 2S1^5 - 5S1^6 - 5S1^7 - S1^8]$$
and the [ ] can be calculated

3. Hence $q_{100} = ((350 - 60)q_{99} - K)/K1$