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Singapore Actuarial Society

One Year VaR and Real World Projections

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Agenda

- Insurance Risk Management Simulation Framework
- One Year VaR Capital
  - Nested Stochastic
  - Covariance Matrix
  - Curve Fitting
  - Replicating Portfolio
  - Least Square Monte Carlo
- Real World Projection: Key Questions, Challenges and Solutions
  - Point in Time vs. Through the Cycle
  - Equity Calibration
  - B+H Calibration Process

Insurance Risk Management Simulation Framework

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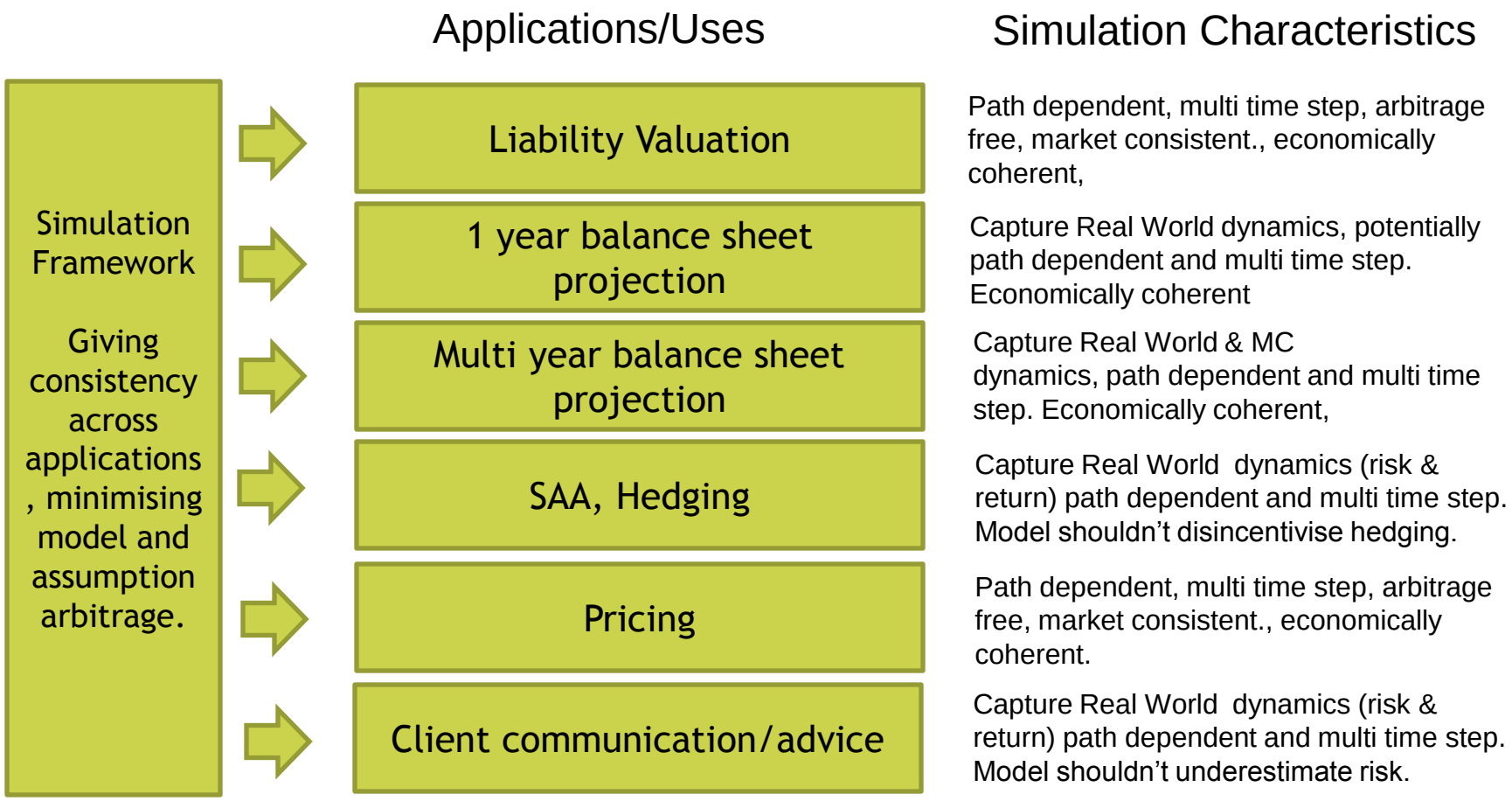
Establishing A Simulation Framework



- The main danger is that multiple frameworks are being used, with conflicting assumptions leading to sub optimal risk management

Simulation Framework - Possible Options

Single Period Model

Statistical distribution of risk factors at a single future point in time.

Usually structured as marginal risk distributions plus copula

- Captures RW dynamics if it can be calibrated.
- Ignores path dependency.
- Cannot be easily scaled to multi time periods.
- Only has one purpose so potentially creates inconsistency.
- **A risk management cul de sac ?**

Multi Period Model no Economic Structure

Multi time step Monte Carlo simulation model with no economic structure.

Each risk factor considered separately plus a dependency structure

- Captures path dependency/multi time steps.
- Captures RW dynamics for individual risk factors
- Can be used for multiple purposes (valuation, balance sheet projection etc)
- Lack of structure may introduce arbitrage and undermine risk management strategies.

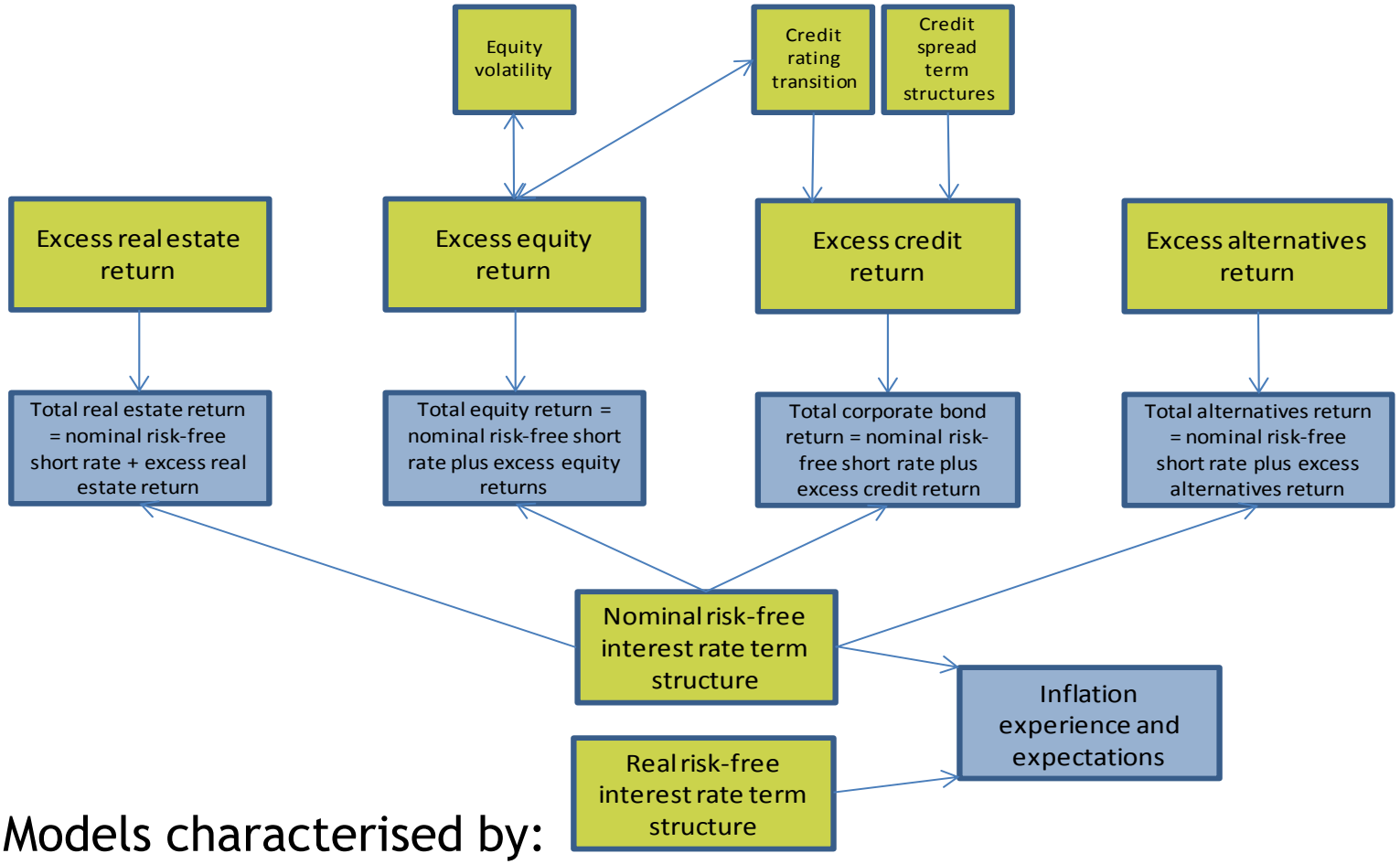
Multi Period Model with Economic Structure

Multi time step Monte Carlo simulation model with an economic structure

The relationships between risk factors are integral to the model

- Captures path dependency/multi time steps.
- Captures RW dynamics for individual risk factors
- Can be used for multiple purposes (valuation, balance sheet projection etc)
- Structure ensures no arbitrage and supports effective risk management strategies.

B+H Basic Single-Economy ESG Structure

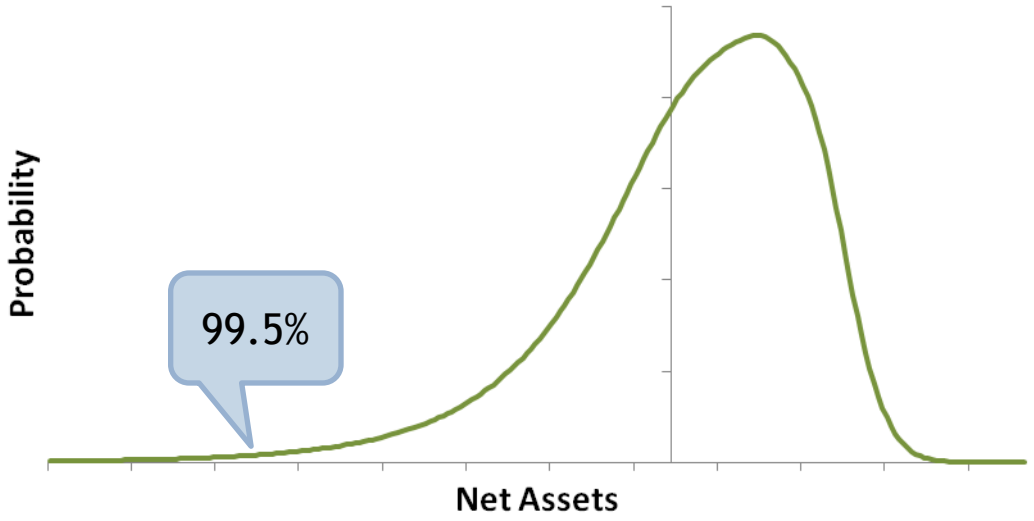


- Models characterised by:
  - Correlation relationships that ensure plausible economic relationship between asset classes
  - Equations that describe the stochastic evolution of key financial assets

One Year VaR Capital

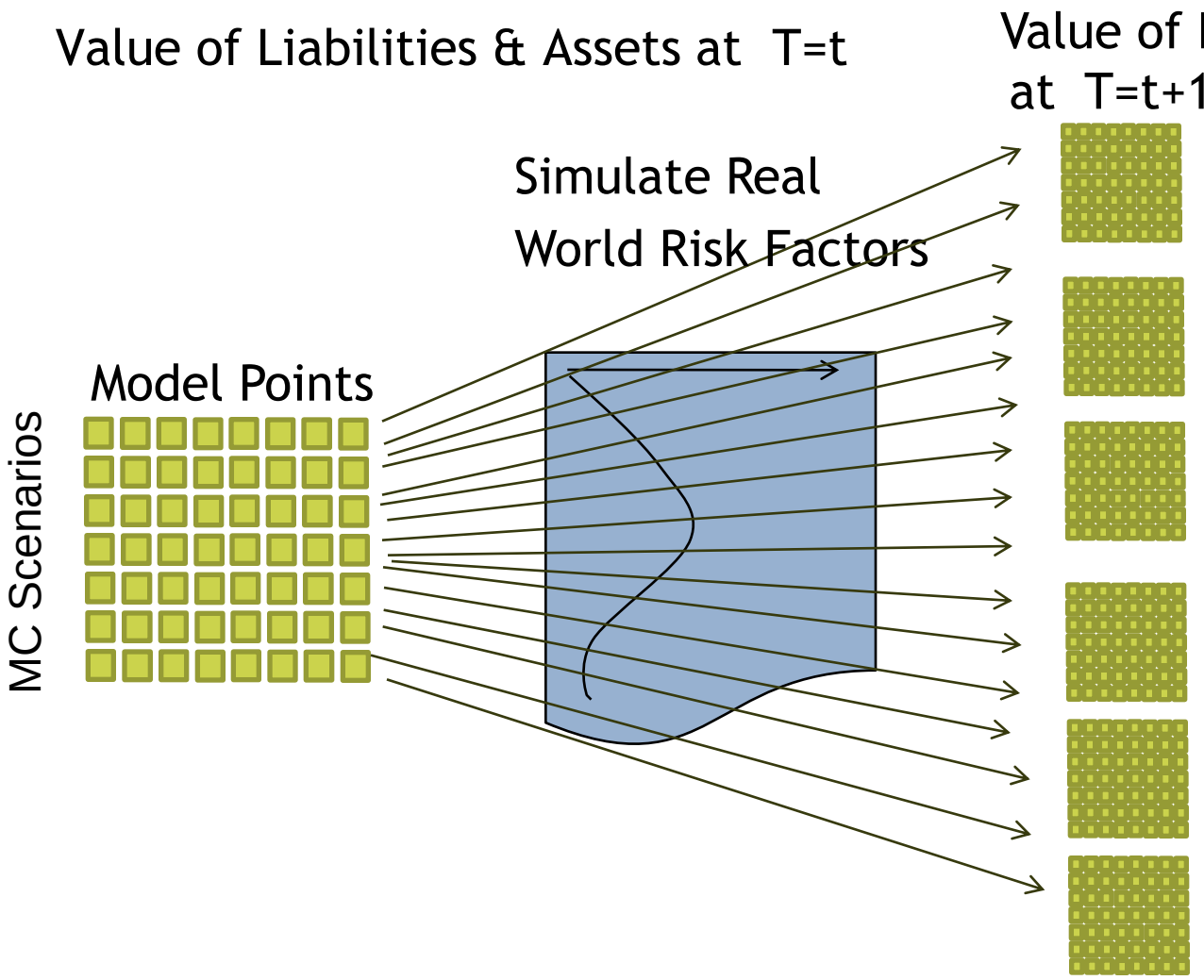
How do we Calculate 1 Year VaR Capital?

- Ideally want to know what the full probability distribution of Net Assets looks like in 1 year (may settle for knowing left hand tail)
- Full Nested Simulation is often required



- Need to know 2 things to get this distribution:
  - Probabilities of economic (and other) events occurring
  - What happens to our balance sheet under these events

Method for Calculating 1 Year VaR EC Full Nested Stochastic simulation



Classic nested simulation problem

1000's of model points
1000's of MC Scenarios
1000's of RW Scenarios

Computationally difficult.

1-year VaR Capital Implementations

- Current 1-year VaR implementations generally use an approximation to the full Stochastic-on-Stochastic approach
- A range of approximation methods have been used, e.g.:
  - Covariance matrix
  - Curve-fitting
  - Least Square Monte Carlo
  - Replicating Portfolios
- Each method offers different trade-offs:
  - Technical accuracy
  - Ease of implementation
  - Transparency of approximation error.
- These trade-offs are generally not well understood.

Covariance Matrix

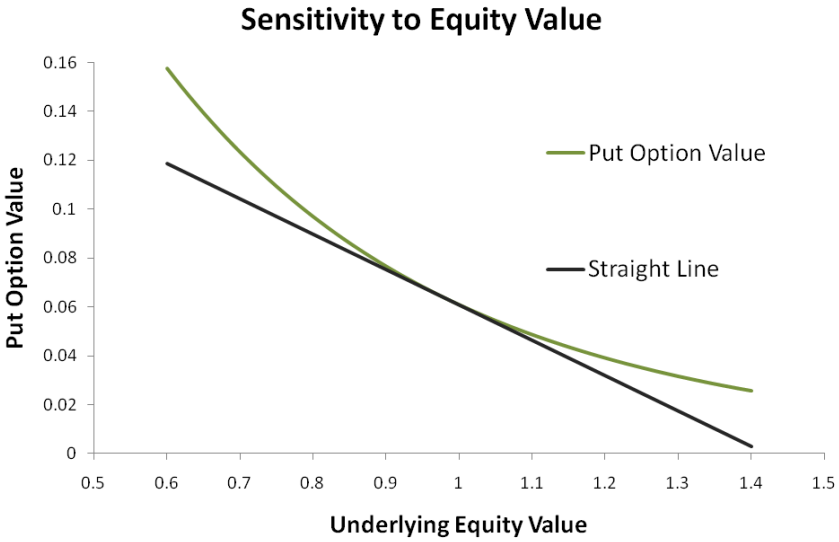
- Determine the 99.5% instantaneous shock for each individual risk
- Calculate change in balance sheet for each risk
- Aggregate capital requirement = “root sum squares”

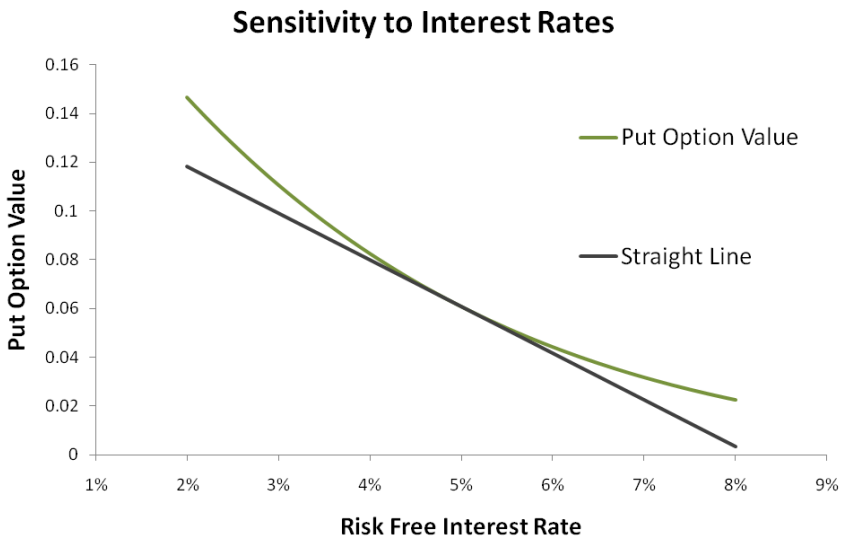
$$SCR_{market} = \sqrt{\sum_{i,j} Corr_{i,j} \times SCR_i \times SCR_j}$$

- Assume that risks follow a multivariate normal (elliptical) distribution
  - Dependence is linear
  - Individual risks are linear

Non-Linear Liabilities

- Liabilities are non-linear under changes in many risk drivers
- Put option example:

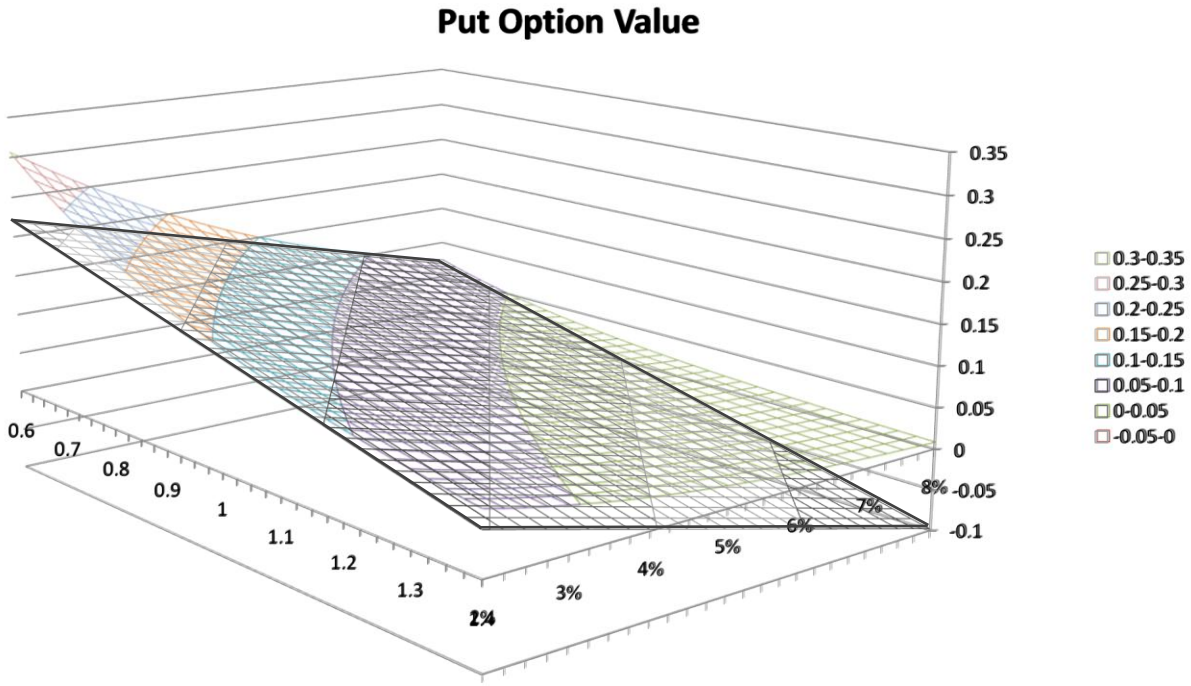




- Value of liability increases by more than a linear model suggests
  - For both equity and interest rates

Covariance Matrix Implicit Assumption

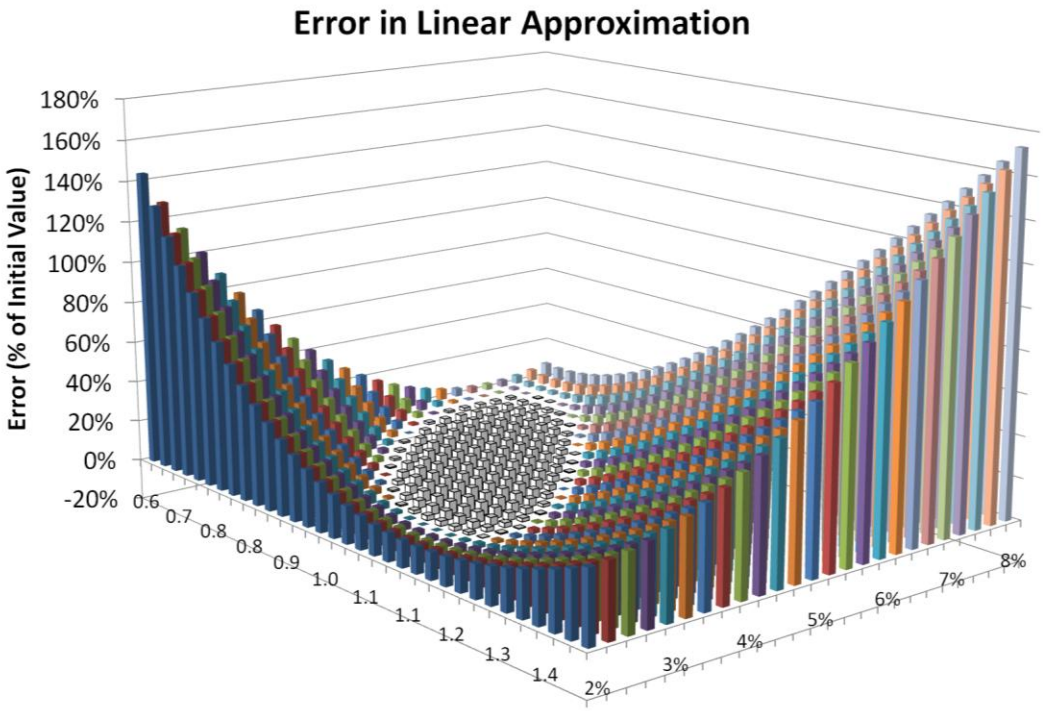
- Value is individually and jointly linear
- Only correct in some places



- Tend to underestimate capital when liabilities are non-linear

Error of Linear Approximation

- Error is potentially larger than original liability



- Correct for a ring in the centre
  - This area does not correspond to area where joint behaviour is important
  - Important area is determined by correlation

Covariance Matrix

- Used in standard formula
- Glaring and substantial flaws
- High level of inaccuracy relative to detailed calculations done elsewhere
- May try to adjust results to take account of errors
  - Increase correlations
  - Non-linearity scaling factor
  - “Fixes” are generally inaccurate and do not work well together

Curve Fitting

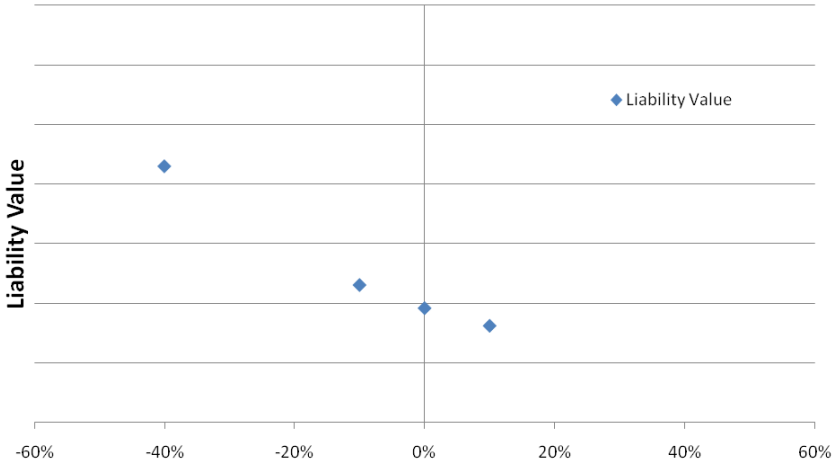
- Best solution would be to do nested stochastic simulation
  - This would give us a very accurate estimate of the balance sheet function
  - But is very time consuming
- A compromise is to approximate the change in balance sheet with a formula
  - Perform a number of instantaneous shocks
  - Fit a multi-dimensional formula to instantaneous shocks
- Use Monte Carlo simulation to get distribution of economic (and other) variables
  - Use formula to approximate balance sheet in each simulation

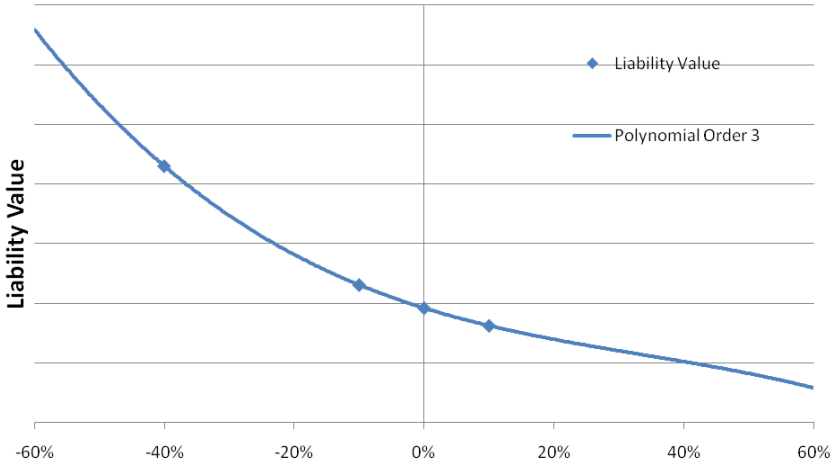
$$L_i(t_1) = a_0 + a_1 V_i(t_1) + a_2 V_i(t_1)^2 + a_3 S_i(t_1) + a_4 S_i(t_1)^2 + a_5 V_i(t_1) S_i(t_1)$$

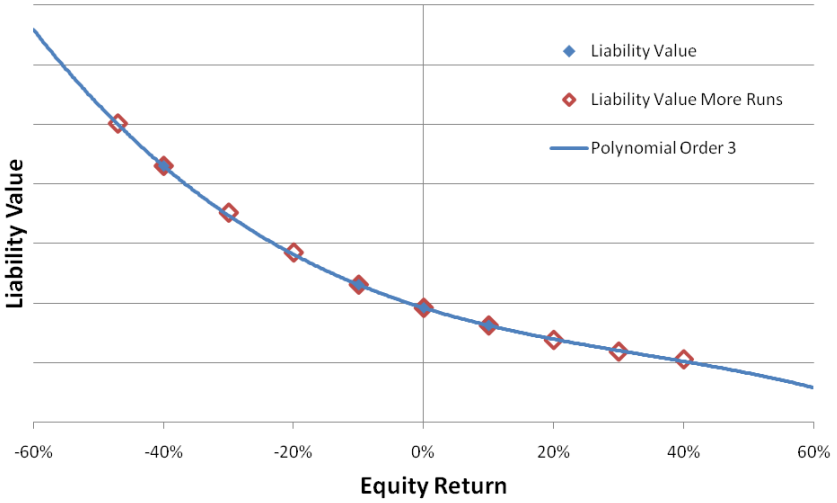
- Calculate capital required from distribution of balance sheet

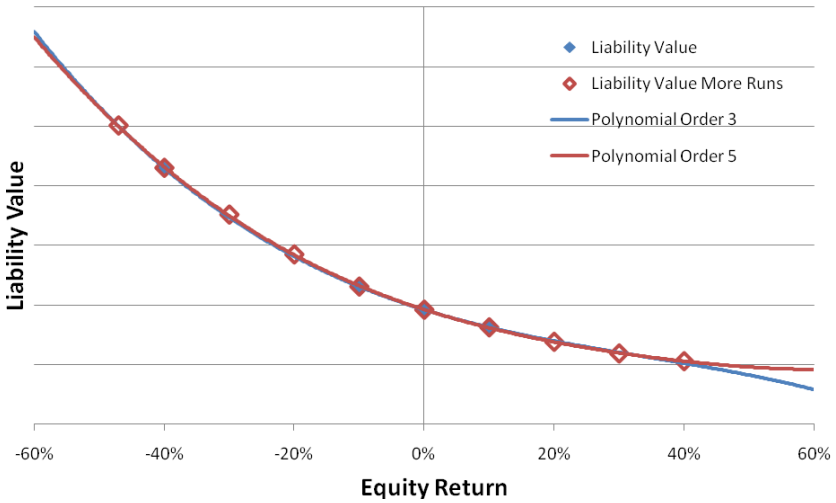
Curve Fitting

- Increased accuracy with more stresses and cross stresses



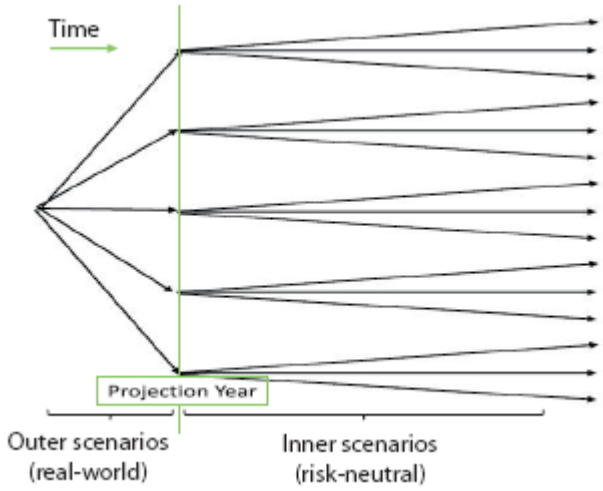


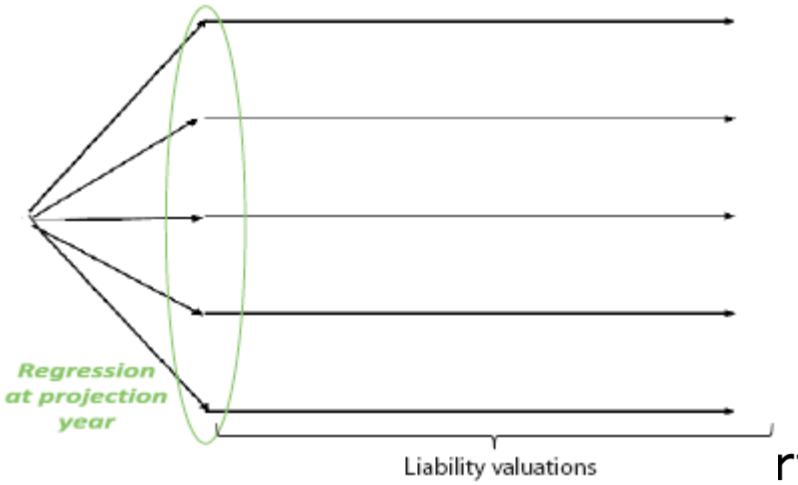




Least Squares Monte Carlo

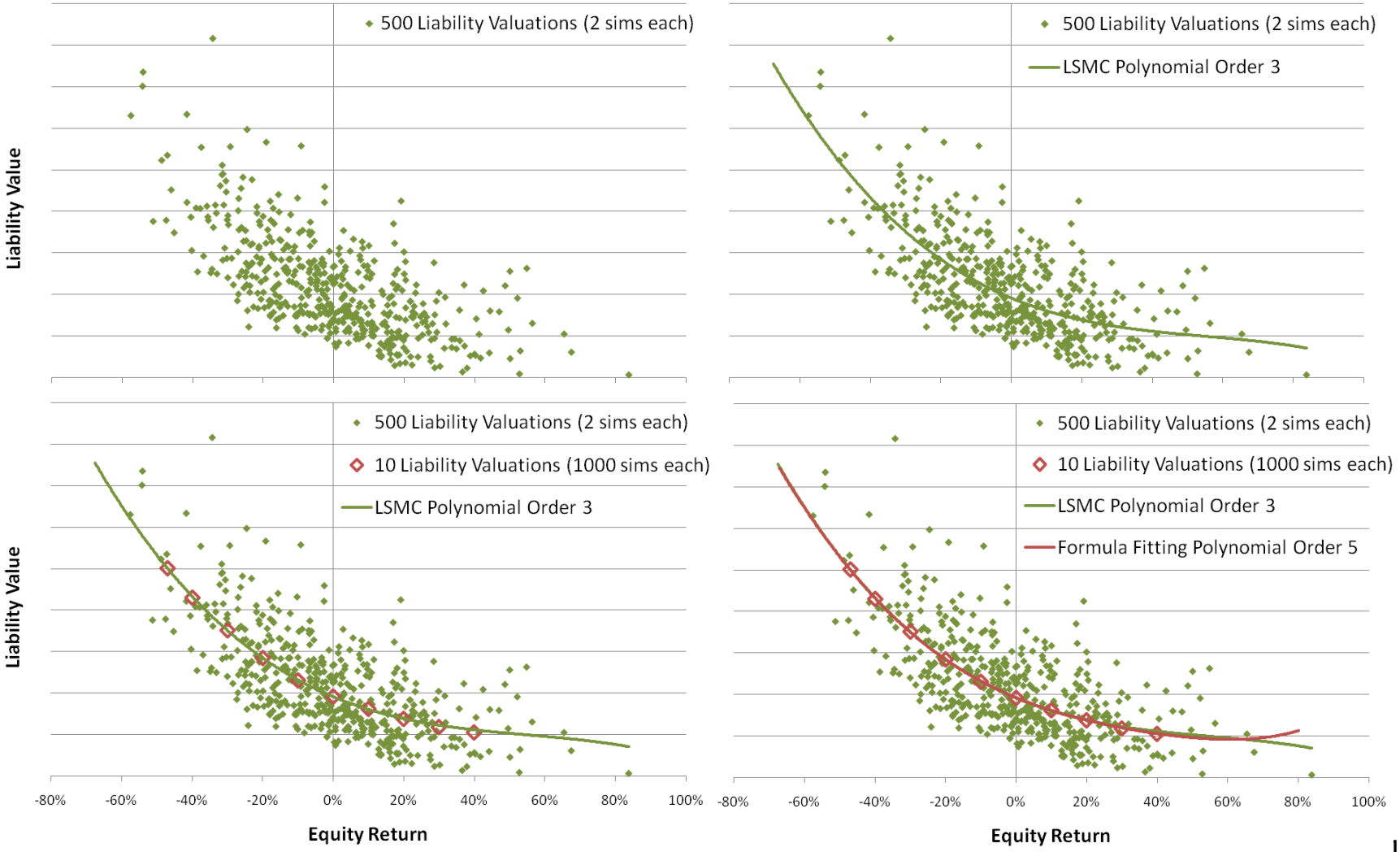
- Technique used in derivative pricing for American / Bermudan options
- B&H has applied to assessment of SCR for complex insurance liabilities
- Instead of doing full nested simulation, only do 1 inner simulation
- Then fit the parameters of the specified approximating liability valuation function by least squares minimization of differences in valuation simulation results and the valuation function results for each scenario





Least Squares Monte Carlo

- With profits book



Least Square Monte Carlo

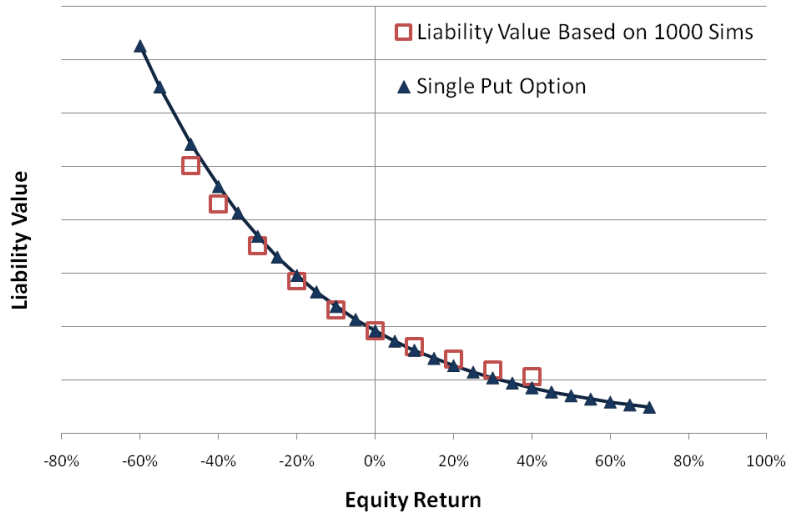
- Fast (only 1 run) and more efficient than curve fitting
- Method extends well to many risk dimensions and types of liabilities
- Possible to estimate potential error in estimate of SCR
- Improve accuracy with number of inner simulations
- Can improve fit with tailored set of outer scenarios (eg get more points in the tails of the function to improve fit)

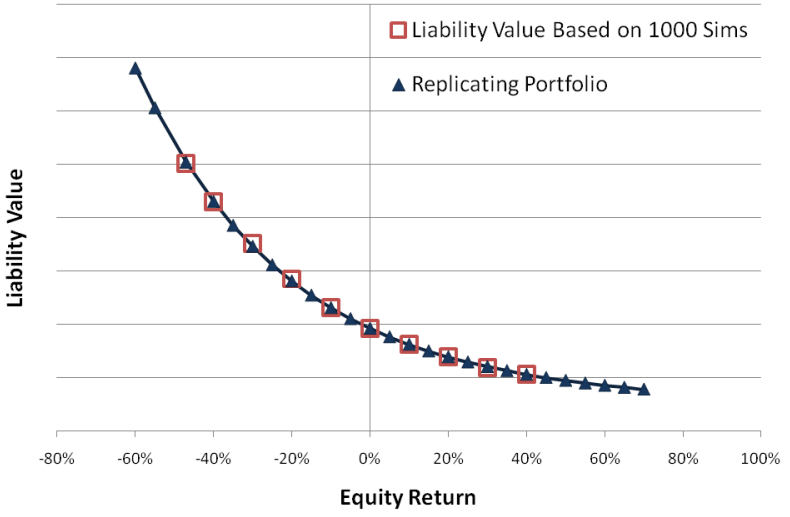
Replicating Portfolios

- Think of our Liability book like a portfolio of assets
- Include assets that behave in a similar way to liabilities
  - Bonds
  - Equities
  - Options
  - Swaptions
  - etc
- Generate the cashflows from liabilities and assets in a Monte Carlo run
- Optimize the weights of the assets so that the best fit is achieved
- Can fit to PV or individual years' cashflows
  - PV fitting fine for most capital applications

Replicating Portfolios

- For capital purposes interested in how well Replicating Portfolio fits Instantaneous Shocks
  - Individual and joint
  - Good selection of assets creates better joint behaviour over risks





Replicating Portfolios

- Theoretically appealing to use actual assets in fitting process
- Lends well to economic interpretation and communication across departments
- Gain insight from make-up and change in portfolio over time
- Good extrapolation behaviour
- Can be used in conjunction with other methods

Review

Method	Speed	Economic Probabilities	Balance Sheet Approximation	Economic Interpretation
Covariance Matrix				
Formula Fitting				
Least Squares Monte Carlo				
Replicating Portfolios				
Nested Stochastic				

Observations on Methods (1)

- Although Covariance Matrix has been widely used, now a general understanding of technical and practical limitations.
- Early adopters have gone for an RP approach. They tend to be global businesses with multiple product lines across multiple economies so the driver has not been accuracy but getting a single view of risk exposures across the business.
- Some regional variations, number of French insurers looking to implement a full SoS or LSMC to avoid being in a risk mgmt dead end.
- Number of UK insurers implementing multiple methods depending on liability book eg RP for relatively simple books plus LSMC/Curve Fit for more complex books.

Observations on Methods (2)

- No right answer depends on purpose, danger of being boxed in to a vendor or academic led solution.
- Important to decide if this about just a 1 year VaR approximation or whether it is part of a broader risk management solution. A lot of money can be spent on a risk management dead end.
- All methods are heavily scenario dependant both in the setup and in actual use. Limited expertise in the market in understanding the scenario requirements B&H can work with you to ensure you get an efficient outcome.
- B&H expertise can be used to support/consultant on comparing the various methods, no vested interested and in depth expertise.

Real World Projections: Key Questions, Challenges and Solutions

Different applications bring different challenges

- 1-Year VaR capital
  - 1-year horizon
  - Key questions & challenges
    - 1-in-200-year event, or 1-in-200 event in *coming* year?
    - Consistency with market pricing and risk management incentives?
    - Making plausible assumptions for distributions - including tail events
- Run-off capital
  - Multi-year horizon
  - Key questions & challenges
    - Risk-factor distributions, expected returns and volatility over very long horizons?
    - Importance of economic structure in scenarios
- SAA/Risk Communication
  - Similar to Run-off, but clear focus on asset returns

- *Today we will focus on 1-Year VaR*

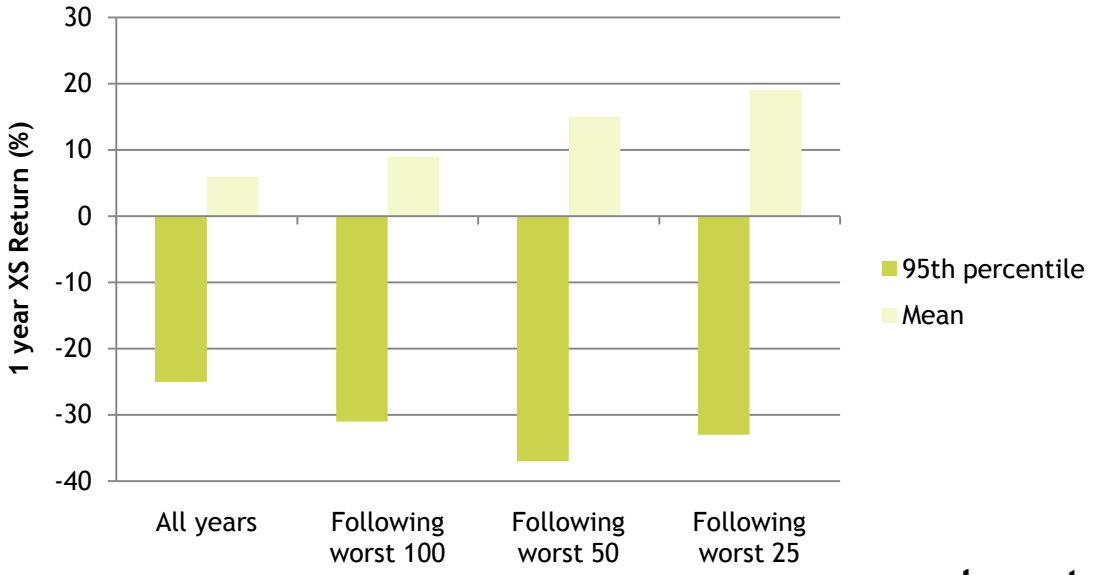
Focus: RW Projection for 1-Year VaR

- Key Question: what is EC approach aiming to achieve?
  - Hold capital against 1-in-200-year event, or 1-in-200 event in coming year?
    - *eg, should we use our forecast for equity volatility over the coming year (Point-in-time) - given current market conditions? Or should we use a long-term (Through-the-cycle) volatility estimate that ignores today's market conditions?*
- Answer sounds obvious?
  - But SII papers suggest that regulators wish to avoid PIT approach
    - Procyclicality..?
- Choice of appropriate calibration methods depends crucially upon answer to this question. Worth exploring…

Comparison of PIT & TTC approaches

- Impact of PIT on capital requirements following crash?
  1. **Vol increases**
     - Sep 2008, B+H PIT estimate of 1-year UK equity vol: 25%
     - Sep 2008, B+H TTC estimate of UK equity vol: 17%
  2. **Mean returns tend to increase**

- Empirically:
  - Following a crash, higher mean returns but vol increase dominates so *tails worse than typical* (16 markets, 100+ years data)

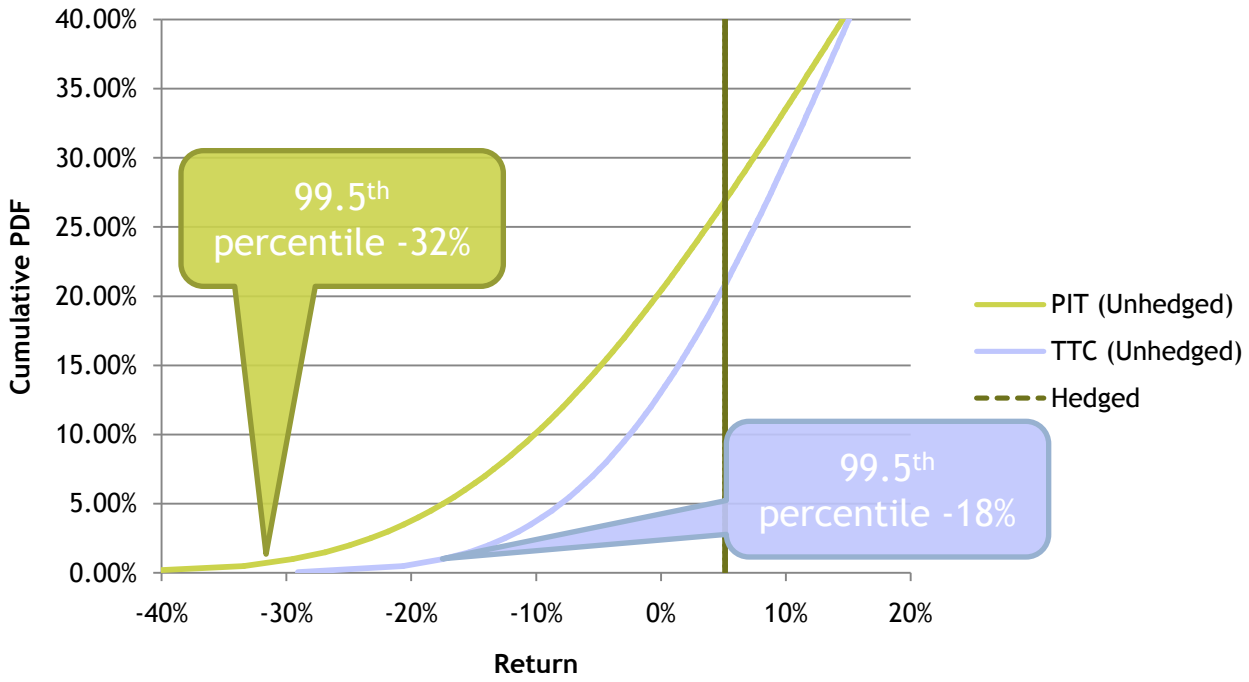


Comparison of PIT & TTC approaches

- So PIT approach likely to *increase capital requirements following a poor year*
  - Procyclicality?
- Current Solvency II aiming to avoid procyclicality
  - Standard formula ~ consistent with TTC approach
    - Equity stress adjustment the exception
- But is this approach coherent?
  - Disconnect between today's *market prices* and *expectations*
    - Discourages capital management….?

TTC Pitfalls? An Example

- Simple problem:
  - Your company has single liability (ATM put option) and some cash. Do you hedge the put?



  - Assumptions: Equity returns assumed lognormal with mean-return proportional to assumed volatility; option IV = 120% of PIT vol estimate; Assets of ~ 10 times put-value at outset

TTC Pitfalls?

- TTC approach *knowingly* uses forward looking distributions that differ from our 'best guess'
  - Results in disconnect between market prices (*eg* cost of the hedging put option) and assumptions used in projection
  - Disincentive to hedge equity exposure when capital is calculated on 17% vol whilst hedging cost *based on* estimate of 25% vol

- *Potential for PIT and TTC approaches to motivate different risk-management decisions*
  - TTC provides poor platform for economic risk management?
  - SII Use Test

- Better to rely on regulatory flexibility/capital management?
  - ie, not procyclical if firms hedge market risk

RW Projection for 1-Year VaR: Further Challenges

- Beyond this fundamental question…

- Aim is to estimate distribution of the balance-sheet, and in particular the tail position. Requires plausible assumptions for distributions of all relevant assets and risk-drivers, *including tail positions*.

- Challenge: make a consistent, plausible set of assumptions across all risk-drivers

  - What are plausible assumptions for asset returns distribution: mean, volatility, and tail position?

  - … across both *single variables* and *joint* distributions across multiple risks?

Why are these difficult questions?

1. We are dealing with a socio-economic system so that price changes depend upon:
   - Fundamental properties of assets
   - Information: the type / quantity / size of 'news' / shocks that bombard asset markets
   - Psychology!
     + Financial modelling is not physics – markets are not stationary, well-behaved, stable systems.
2. Lack of long-term empirical data
   - 'New' asset classes
   - Extreme events occur rarely (!) i.e. they are "undersampled" in the data we have available (or they may not have been observed at all)
   - The further into the tail, the less confidence we have in estimates
   - Big computers and clever quants don't remove the problem

# years	*Prob of observing at least one 1-in-200 year event*
1	0.5%
10	5%
50	22%
100	39%
200	63%

Constructing Equity Market Assumptions for 1-Year VaR

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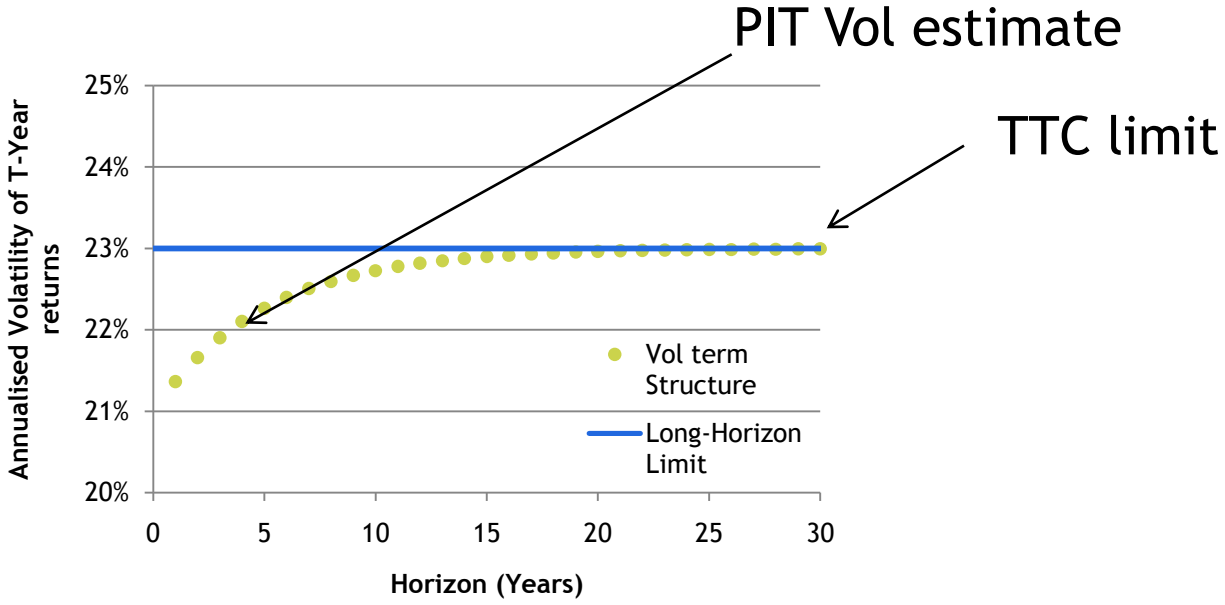
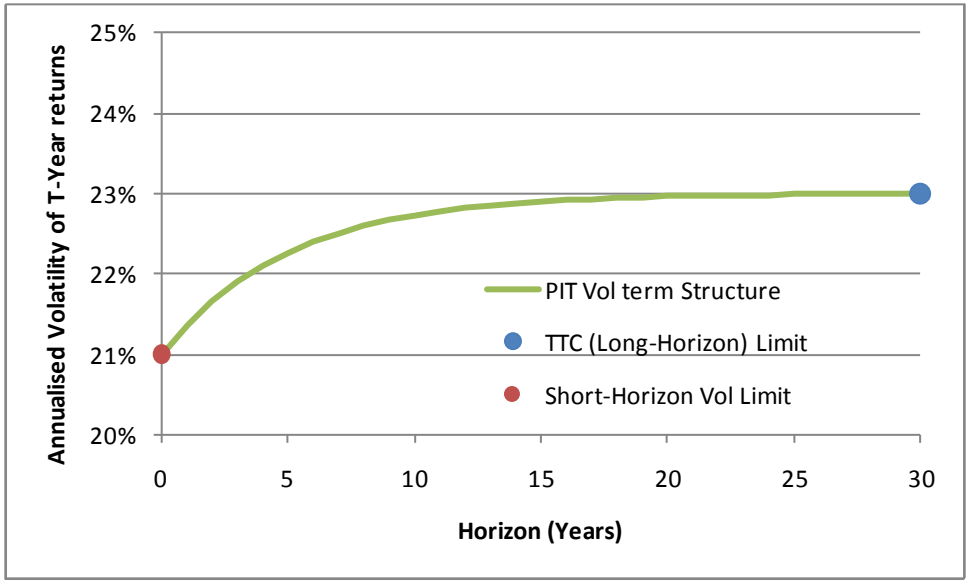
Estimating Equity Market *Volatility*

- Challenge: estimate volatility of equity index for forward projection
  - TTC approach? Make estimate of very long horizon equity volatility
  - PIT approach? Vol can be expected to 'mean revert' towards Long Horizon (TTC) limit. Need to estimate volatility 'term structure'
  - eg, construct an assumption for vol of Nikkei returns

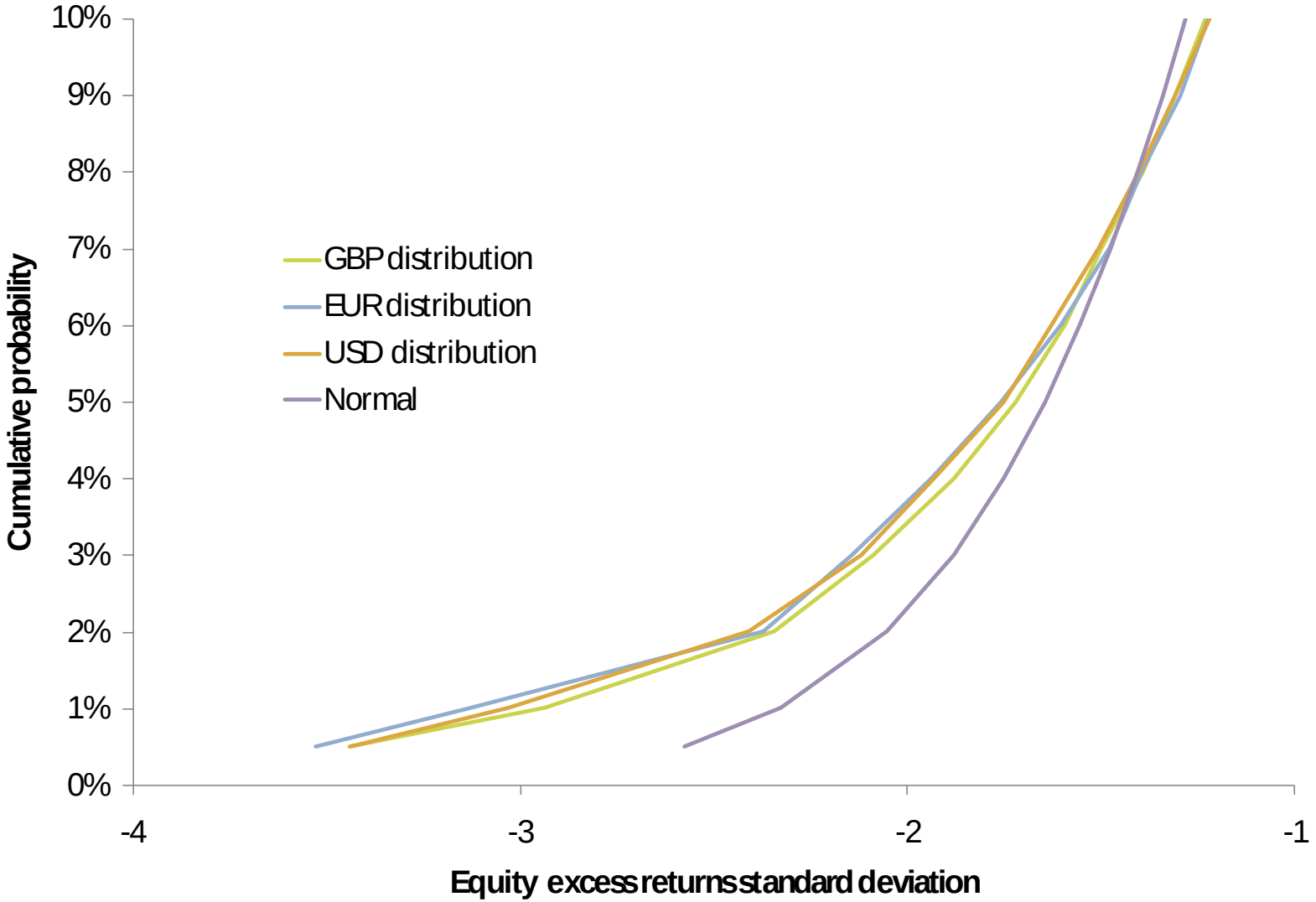


Estimating Equity Market *Volatility*

- Estimate Short-Term Volatility
- Estimate Long-Term (TTC) Volatility
- 'Join the dots'



1 year tail of equity XS-log return distribution



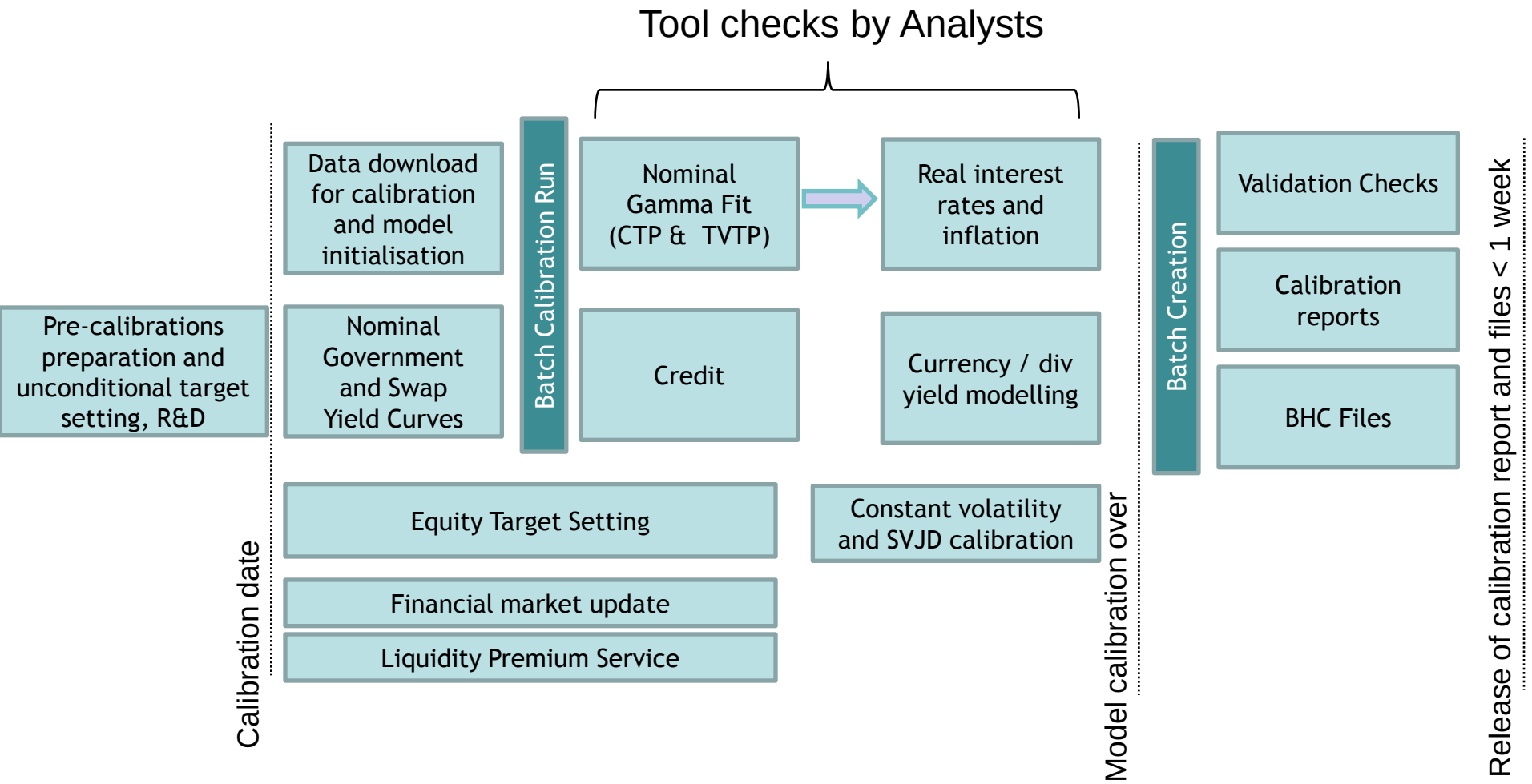
- More skew and kurtosis relative to standard normal distribution

Equity Model Calibration for VaR: Summary

- A considerable amount of research goes into the construction of assumptions for a single asset class
  - Mean Returns
  - Volatility
  - Tail positions…
- We believe that RW sophisticated modeling is key for firms in implementing RM system
  - Simplistic approaches can lead to poor analysis and poor risk management

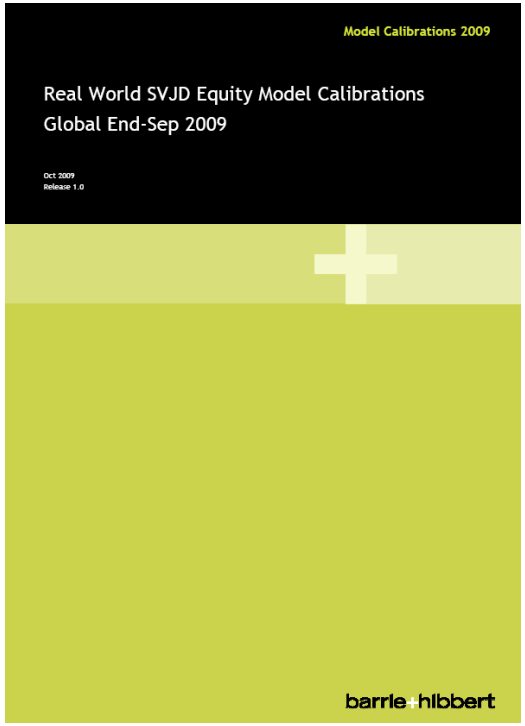
B+H Model Calibration Process

RW calibration process



Model Calibration Process…cont.

- Methods as described above are determined by Calibration research
- Following this B+H update assumptions on regular basis, following these methods
- Models then calibrated to best-match assumptions
  - Quarterly calibration updates
  - Model calibrations documented and ESG model ‘calibration files’ produced
    - *eg*



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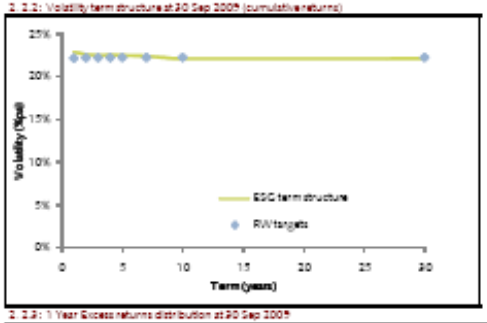
2.2 SVJD Equity Model - EUR Equity

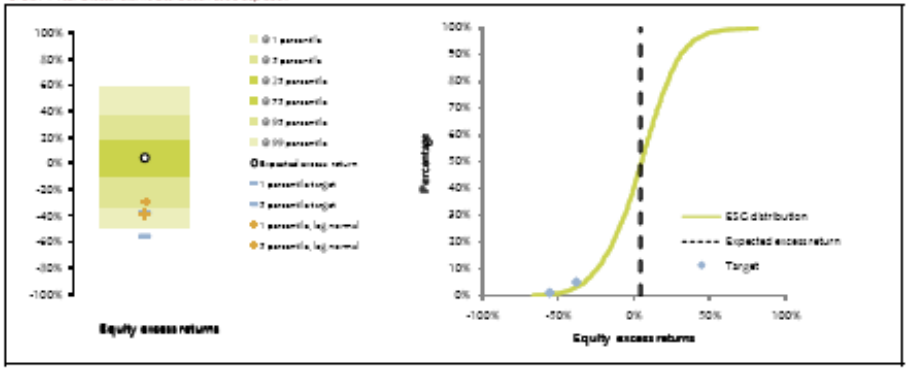
2.2.1: Model parameters at 30 Sep 2009

		30 Sep 2009	30 Jun 2009
	Model:	Equity (SVJD Model)	
	Equity Asset:	EUR Equity	
	Calibrated To:	RW Volatility Targets	
	Excess return mean	0.0446	0.0447
Variance Model	Initial value	0.0162	0.0159
	Volatility	0.5000	0.5000
	Mean Reversion Rate	2.7565	2.7565
	Mean Reversion Level	0.0045	0.0045
	Correlation	-0.5300	-0.5300
Jump Model	Jump Mean	0.0000	0.0000
	Arrival Rate	0.0000	0.0000
	Jump Volatility	0.0000	0.0000
Factor Model	Factor loading 1	1.214	1.257
	Factor loading 2	-0.220	-0.205
	Factor loading 3	-0.031	-0.027
	Factor loading 4	-0.102	-0.161
	Factor loading 5	-0.288	-0.265
	Factor loading 6	0.000	0.000
	Income yield	0.0405	0.0506
	(DJEUROSTOXX50)	Source: Bloomberg	

2.2.2: Volatility term structure at 30 Sep 2009 (cumulative returns)



2.2.3: 1 Year Excess returns distribution at 30 Sep 2009



Commentary

The parameters presented in exhibit 2.2.1 are for the asset specific SVJD model for both 30 Sep 2009 and the previous quarter.

In exhibit 2.2.2 we present the term structure of excess cumulative return volatility (%pa). Also shown are our target volatilities at 1-5, 7, 10 and 30 year horizons.

The 1 year excess return distribution is shown in exhibit 2.2.3. Here we present the downside percentile targets and the percentiles of a log-normal excess return distribution with equivalent mean and volatility. Note, for display purposes only, we cap the upside of the distribution at 100%.

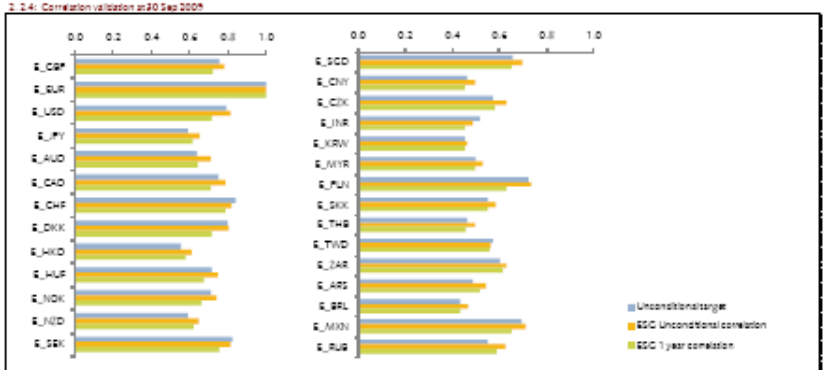
In calibrating SVJD we target explicitly target the skew and kurtosis of the instantaneous unconditional 1 month distribution. We expect, however, to be close to the 1 year percentile targets even though they are not targeted explicitly. In general the downside percentiles will exhibit fatter tails than the equivalent log-normal distribution.

The equity-equity correlations are presented in exhibit 2.2.4. The model is calibrated to target unconditional correlations (i.e correlations at long time horizons). Over shorter horizon the correlation between equity markets will vary. The precise change in correlation is driven by the relative proportions of specific and systematic volatility for each asset, which is partially specified by the relative factor loadings of the equity asset and the conditional stochastic variation in the factor 1 volatility.

A key feature of the SVJD model is the tail dependence exhibited by the joint equity excess return distributions. Tail dependence is the probability that an extreme tail event occurs in both economies. We present the tail dependence for 95%, 99% and 99.5% percentile tail events. In addition we show the level of tail dependence seen in a bivariate log-normal excess return distributions with correlation of 0.6 (average global developed market correlation). In many instance the level of tail dependence is higher for our SVJD model calibration relative to a log normal excess return distributions of similar correlation.

2.2 SVJD Equity Model - EUR Equity

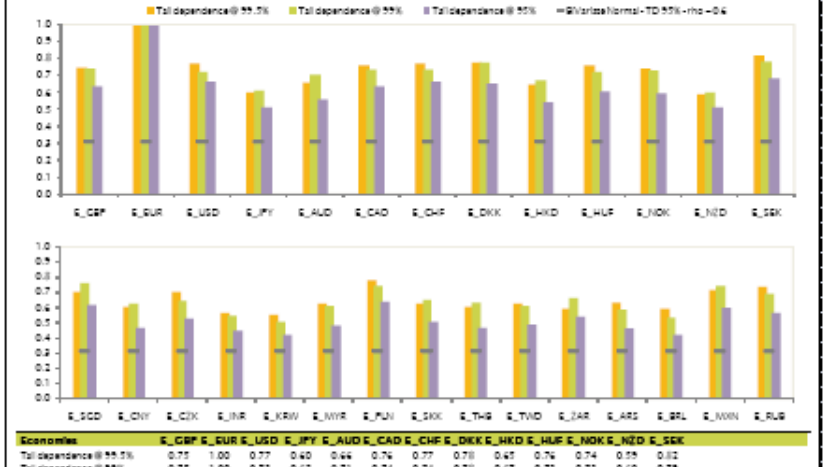
2.2.4: Correlation validation at 30 Sep 2009



Economies	E_GBP	E_EUR	E_USD	E_JPY	E_AUD	E_CAD	E_CHF	E_DKK	E_HKD	E_HUF	E_NOK	E_NZD	E_SEK
Unconditional target	0.74	1.00	0.75	0.55	0.64	0.75	0.84	0.80	0.55	0.72	0.71	0.59	0.82
ESC Unconditional correlation	0.73	1.00	0.82	0.45	0.71	0.79	0.82	0.80	0.47	0.75	0.74	0.45	0.81
ESC 1 year correlation	0.72	1.00	0.72	0.62	0.64	0.71	0.79	0.72	0.58	0.67	0.66	0.62	0.76

Economies	E_SGD	E_CNY	E_CZK	E_INR	E_KRW	E_MYR	E_PLN	E_SKK	E_THB	E_TWD	E_ZAR	E_ARS	E_BRL	E_MXN	E_RUB
Unconditional target	0.66	0.46	0.57	0.52	0.46	0.50	0.73	0.55	0.47	0.57	0.60	0.45	0.44	0.70	0.55
ESC Unconditional correlation	0.70	0.50	0.63	0.45	0.46	0.52	0.74	0.58	0.50	0.57	0.62	0.55	0.47	0.71	0.62
ESC 1 year correlation	0.65	0.46	0.58	0.46	0.46	0.50	0.63	0.55	0.46	0.56	0.62	0.52	0.48	0.67	0.59

2.2.5: Tail dependence at 1 year at 30 Sep 2009



Economies	E_GBP	E_EUR	E_USD	E_JPY	E_AUD	E_CAD	E_CHF	E_DKK	E_HKD	E_HUF	E_NOK	E_NZD	E_SEK
Tail dependence @ 99.5%	0.75	1.00	0.77	0.60	0.66	0.76	0.77	0.70	0.67	0.76	0.74	0.59	0.82
Tail dependence @ 99%	0.75	1.00	0.73	0.62	0.71	0.74	0.74	0.70	0.67	0.72	0.72	0.60	0.75
Tail dependence @ 95%	0.64	1.00	0.67	0.52	0.56	0.64	0.67	0.65	0.55	0.61	0.60	0.51	0.68

Economies	E_SGD	E_CNY	E_CZK	E_INR	E_KRW	E_MYR	E_PLN	E_SKK	E_THB	E_TWD	E_ZAR	E_ARS	E_BRL	E_MXN	E_RUB
Tail dependence @ 99.5%	0.70	0.60	0.70	0.56	0.55	0.62	0.70	0.62	0.60	0.62	0.59	0.62	0.59	0.71	0.72
Tail dependence @ 99%	0.76	0.62	0.64	0.55	0.50	0.61	0.74	0.65	0.60	0.61	0.66	0.58	0.58	0.74	0.69
Tail dependence @ 95%	0.61	0.47	0.52	0.44	0.42	0.48	0.63	0.50	0.46	0.49	0.52	0.46	0.42	0.59	0.56

Model Calibration Process: Coverage

- B+H offer RW VaR Calibrations across:
  - 29 Economies (Interest rates, inflation, credit)
  - 31 equity indices
- Similar coverage for Multi-Year Calibrations
  - Run-off capital calculations
- Economy coverage in detail:
  - GBP EUR USD JPY CHF ZAR AUD CAD PLN THB CZK HUF TWD DKK CNY HKD KRW SGD SEK NOK MYR NZD INR ARS BRL MXN RUB PHP IDR

Summary

- Suitable RW scenarios are key to a useful risk management framework
  - Poor quality analysis/modelling can lead to misleading results with an impact upon RM strategies adopted
    - No ‘right’ answers but some more wrong than others!
  - Important to ask what is the aim of the exercise
    - Model setup that is appropriate for one application may not be appropriate for others
- Significant effort required to provide sophisticated RW models and calibration assumptions
  - B+H dedicate significant expert resource to this problem
  - Provide RW model calibrations for specific purposes

Questions

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