

Aggregation of risks and dependencies

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Agenda

1	Copula a general model for dependence
2	Various forms of copula and their influence on diversification
3	Capital allocation and dependence
4	Hierarchical tree to model a large number of risks
5	Conclusions

Determining Risk-Adjusted Capital

- The risk-adjusted capital (RAC) of an insurance company is evaluated on the basis of a quantitative model of its different risks
- We first need to identify the various sources of risk. One usually distinguishes four large risk categories:
 1. Underwriting risk (or liability risk),
 2. Investment risk (or asset risk),
 3. Credit risk (or risk of default),
 4. Operational risk
- Generally insurance companies have the know-how to manage and model their liability risk and are able to model the next two categories as well using standard finance models

Dependence Between Risks is Key

- **Risk Diversification** reduces a company's need for risk-adjusted capital. This is key to both insurance and investments.
- However, risks are rarely completely independent:
 - Stock market crashes are usually not limited to one stock market.
 - Certain lines of business are affected by economic cycles, like aviation, credit & surety or life insurances.
 - Motor insurance is also correlated to motor liability insurance and both will vary during economic cycles.
 - Big catastrophes can produce claims in various lines of business.
- Dependence between risks **reduces the benefits** of diversification.
- The influence of dependence on the aggregated RAC is thus crucial and needs to be carefully analyzed.

Linear dependence: *Correlation coefficient*

The linear (Pearson) correlation coefficient between two random variables X and Y is defined as:

$$\rho(X, Y) = \frac{\text{COV}(X, Y)}{\sqrt{\sigma^2(X) \sigma^2(Y)}} \in [-1, 1]$$

where $\text{COV}(X, Y)$ is the covariance between X and Y ,

$$\text{COV}(X, Y) = E(XY) - E(X)E(Y)$$

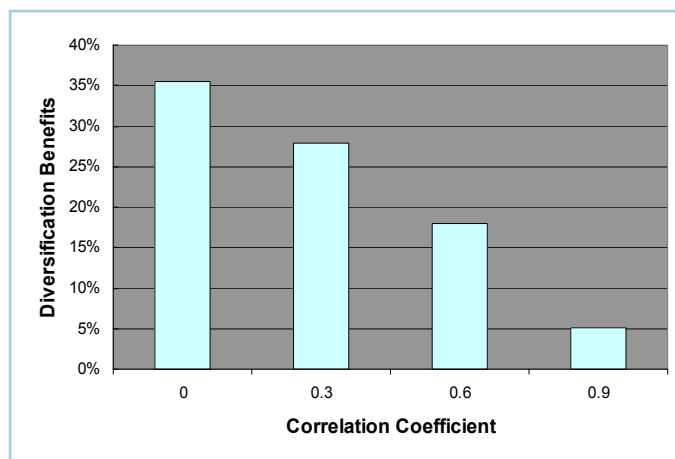
and $\sigma^2(X)$, $\sigma^2(Y)$ denote the variance of X and Y respectively.

Influence of Correlation on RAC

- Let us take the same risk twice (lognormally distributed, $\mu=10$ and $\sigma=1$) and bundle them in a portfolio.
- Let us vary the correlation between the risks from 0 to 0.90.
- Here are the various diversification benefits, D , in percent:

$$D = 1 - \frac{RAC_P}{\sum_i RAC_i}$$

where RAC_P is the portfolio RAC and RAC_i are the RAC's of the various risks.

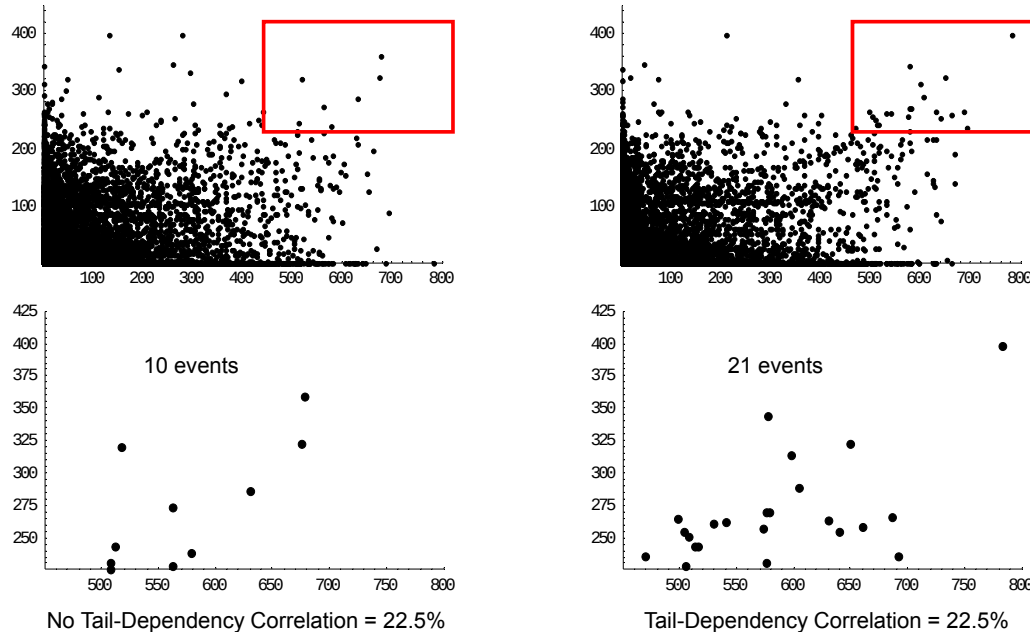


Dependence is not Always Linear

- We have learned to model dependence through **linear correlation**. The whole modern portfolio theory is based on correlation.
- Often dependence increases when diversification is most needed: in case of stress. It is thus **non-linear**.
- It is possible to use the **copulas** instead of linear correlation to model dependences (copula="generalized dependence structure" as opposed to "linear dependence"=correlation).
- The dependence structure will influence greatly the needs for RAC and the diversification benefits one can obtain.

Example: Modelling dependencies with correlations is not enough!

Two joint simulations of losses in two lines of business with various dependence models, but same correlation of 22.5%



Copulas and Sklar's Theorem

- A copula is a multivariate cumulative distribution function $C: [0, 1]^d \rightarrow [0, 1]$ whose margins are standard uniform.
- Uniform margin means:
 $C(1, \dots, 1, u_i, 1, \dots, 1) = u_i$ for all $i \in \{1, \dots, d\}$ and $u_i \in [0, 1]$
- **Sklar's Theorem:** Let F be a joint distribution function with margins $F_1, \dots, F_d$. There exists a copula such that for all $x_1, \dots, x_d$ in $[-\infty, \infty]$,

$$F(x_1, \dots, x_d) = C(F_1(x_1), \dots, F_d(x_d))$$

- If the margins are continuous then C is unique.
- Conversely, if C is a copula and $F_1, \dots, F_d$ are uniform univariate distributions, then F defined above is a multivariate distribution function with marginals $F_1, \dots, F_d$

Example of copulas

- Independence: $X_1, \dots, X_d$ are mutually independent if and only if their copula C satisfies

$$C(u_1, \dots, u_d) = \prod_{i=1}^d u_i$$

- Comonotonicity – perfect dependence: $X_i = T_i(X_1)$ strictly increasing with $i = 2, \dots, d$ if and only if C satisfies

$$C(u_1, \dots, u_d) = \min \{u_1, \dots, u_d\}$$

- Anti-comonotonicity – perfect negative dependence ($d=2$): $X_2 = T(X_1)$ strictly decreasing if and only if C satisfies

$$C(u_1, u_2) = \max \{u_1 + u_2 - 1, 0\}$$

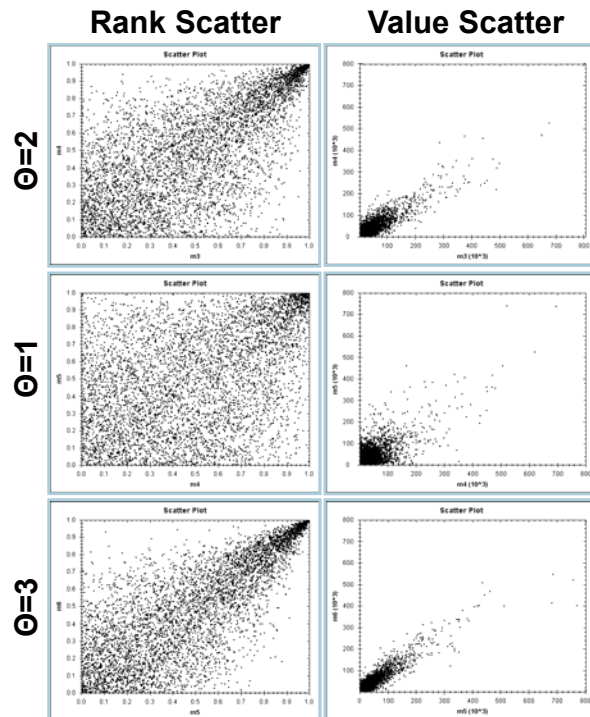
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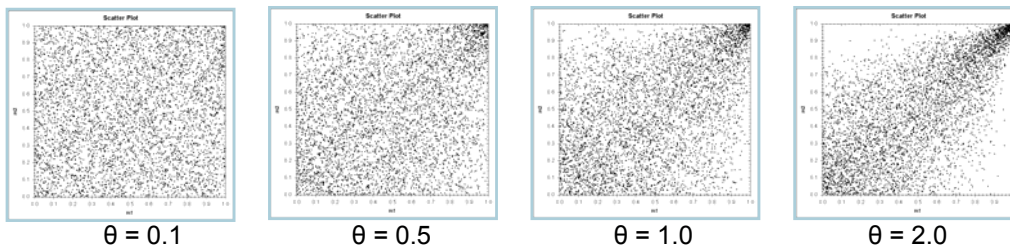
Significance of Value versus Rank Scatter

- Value scatter is used to characterize the spread of the marginal distributions
- Rank scatter shows the underlying dependence between the marginals

For the purpose of analysing dependence we shall display only rank scatter from here on



Archimedean Copula: Clayton Copula



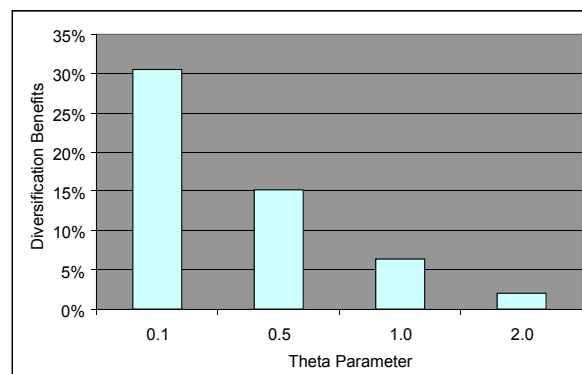
The Clayton Copula CDF is defined by:

$$C(u, v) = \max\left([u^{-\theta} + v^{-\theta} - 1]^{-1/\theta}, 0\right)$$

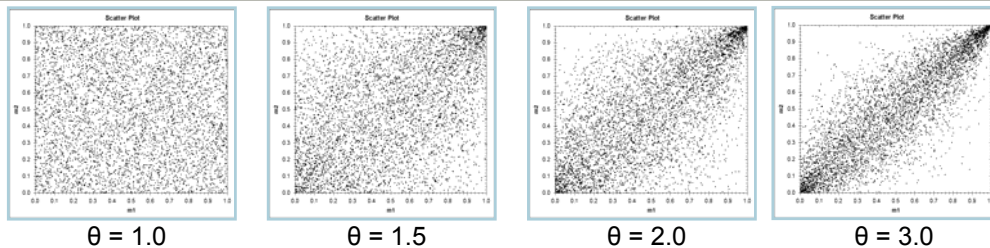
With a *Generator* of the Copula:

$$\phi_{\theta}(t) = \frac{1}{\theta}(t^{-\theta} - 1)$$

The Clayton Copula is Archimedean



Archimedean Copula: Gumbel Copula



The Gumbel Copula CDF is defined by:

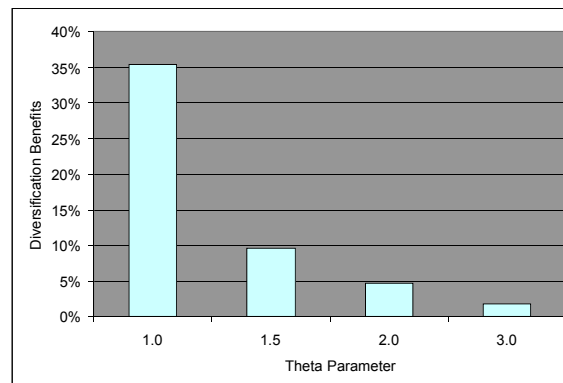
$$C(u, v) = \exp\left(-\left[(-\ln u)^\theta + (-\ln v)^\theta\right]^{1/\theta}\right)$$

$$\theta \in [1, \infty)$$

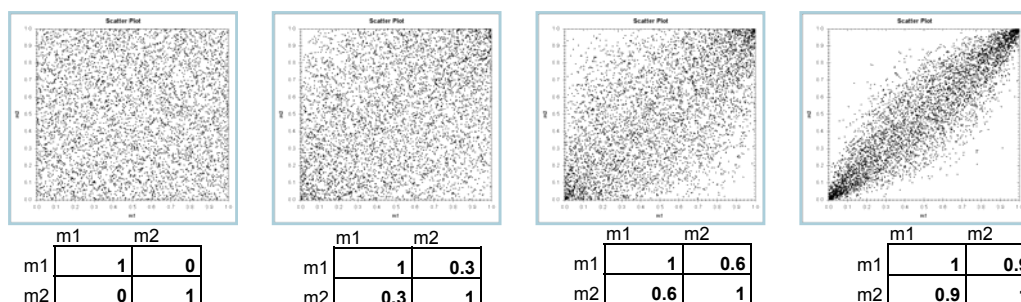
where the *Generator* of the Copula is given by:

$$\phi_\theta(t) = (-\ln t)^\theta$$

The Gumbel Copula is Archimedean



Elliptical Copula: Rank Correlation



The multivariate Normal distribution copula has a matrix as a parameter.
The PDF of a Normal copula is:

$$c_R^{\text{Normale}}(u_1, u_2, \dots, u_n) = \frac{1}{|R|^{\frac{1}{2}}} \cdot \exp\left(-\frac{1}{2} \cdot S' \cdot (R^{-1} - I) \cdot S\right)$$

where, $S_j = \Phi^{-1}(u_j)$, Φ^{-1} is the inverse of the CDF $N(0,1)$ and I is the identity matrix of size n .

The rank correlation is an elliptical copula.

Elliptical Copula: Student's T

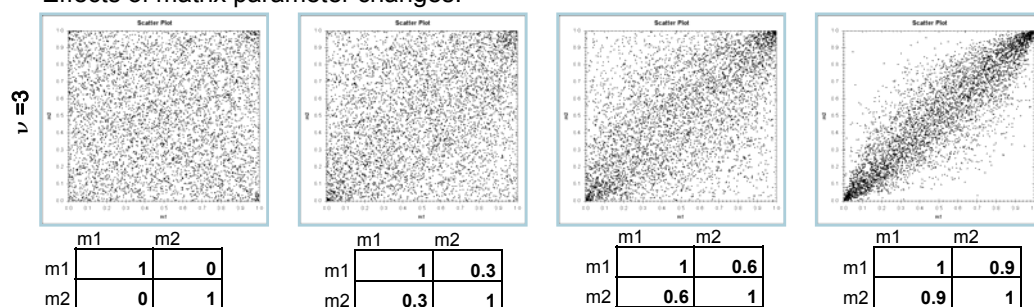
The multivariate Student's T distribution copula also has a matrix as a parameter. The PDF of a Student's T copula is:

$$c_{R,v}^{\text{Student}}(u_1, u_2, \dots, u_n) = |R|^{-\frac{1}{2}} \cdot \frac{\Gamma\left(\frac{v+n}{2}\right)}{\Gamma\left(\frac{v}{2}\right)} \cdot \left(\frac{\Gamma\left(\frac{v}{2}\right)}{\Gamma\left(\frac{v+1}{2}\right)} \right)^n \cdot \frac{\left(1 + \frac{1}{v} \cdot S' \cdot R^{-1} \cdot S\right)^{-\frac{v+n}{2}}}{\prod_{j=1}^n \left(1 + \frac{S_j^2}{v}\right)^{-\frac{v+1}{2}}}$$

Where, $S_j = T_v^{-1}(u_j)$, T_v^{-1} is the inverse of the CDF of the univariate T distribution with ν degrees of freedom

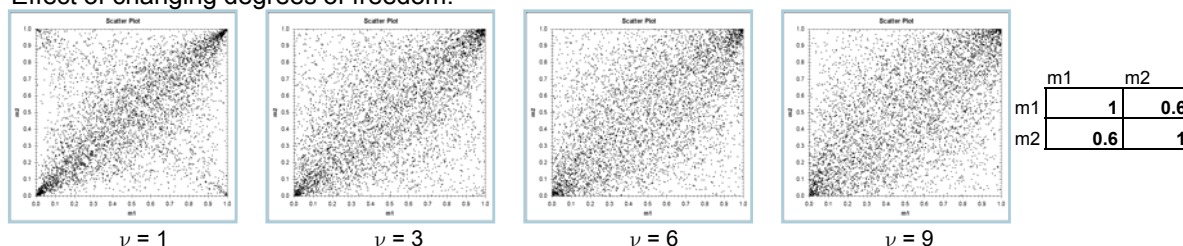
The Student's T copula is an elliptical copula

Effects of matrix parameter changes:

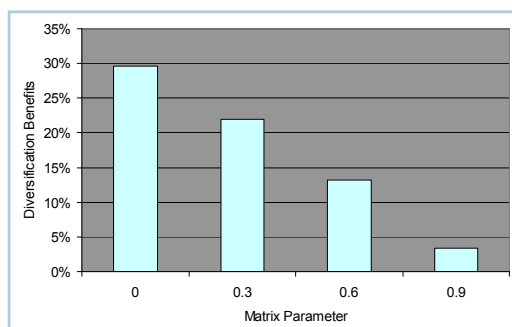


Student's T

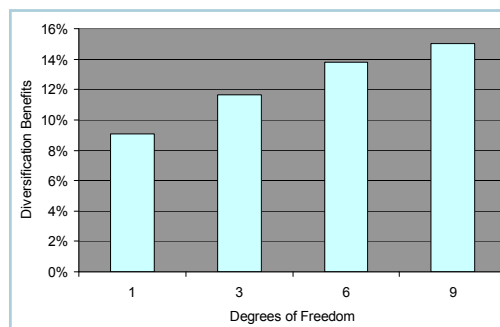
Effect of changing degrees of freedom:



Diversification benefits from varying the matrix parameter (3 degrees of freedom):

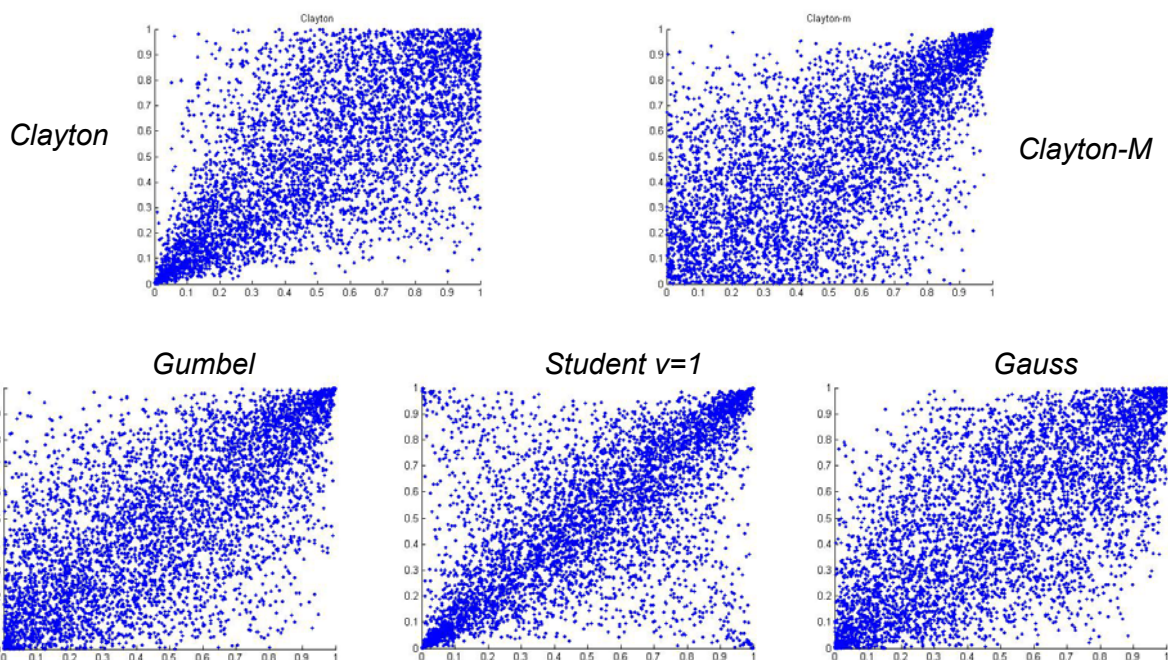


Diversification benefits from varying the degrees of freedom:



Different copulas → different dependencies

Scatter plots with $\rho_\tau=0.5$ (Kendall's tau)



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Allocation formulas

Euler Principle with ES contribution ("Euler")

$$AC_i = E \left[X_i \mid \sum_i X_i \geq VaR_\alpha \left(\sum_i X_i \right) \right] \quad RAC(ES)_i = AC_i - E[X_i]$$

A VaR-based capital allocation principle (NOT coherent) ("VaR I")

$$AK_i = \frac{VaR(X_i)}{VaR \left(\sum_i X_i \right)} \quad RAC(VaR)_i = AK_i \cdot RAC(Var)$$

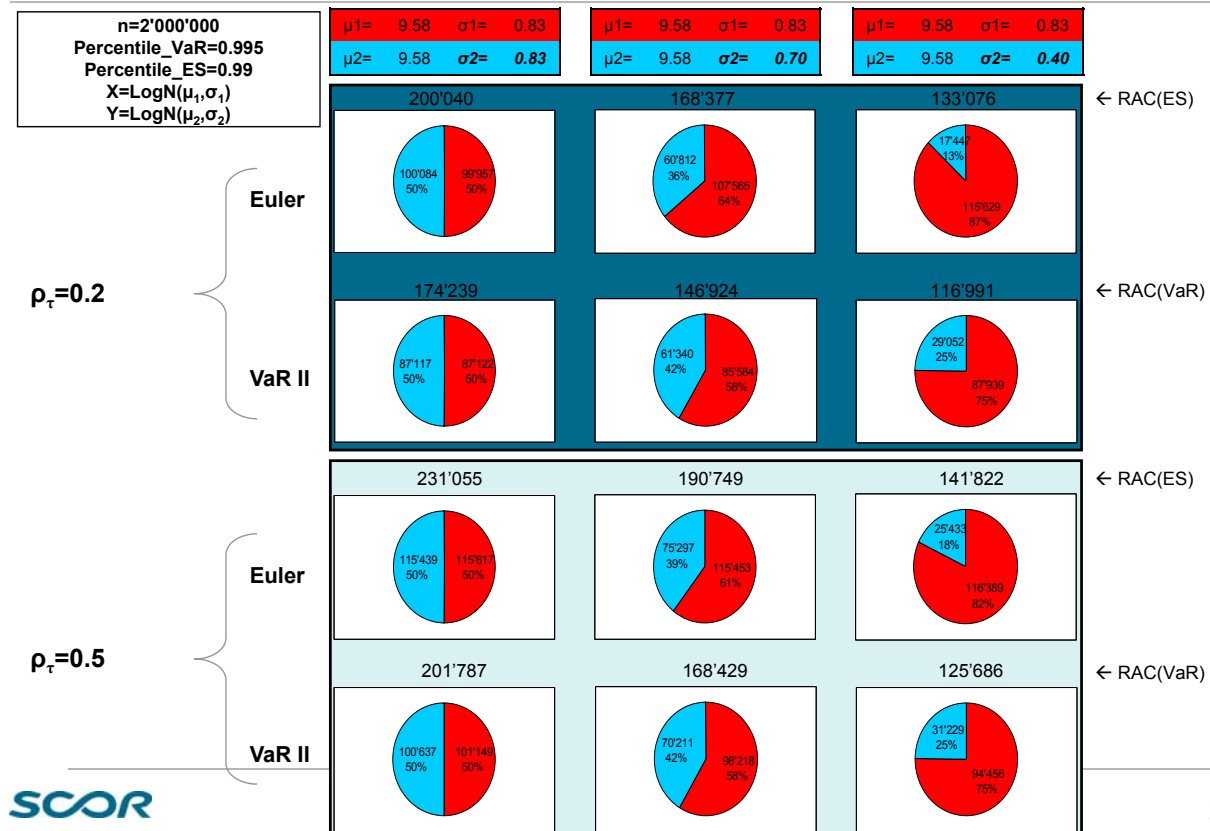
Another VaR-based capital allocation principle ("VaR II")

$$AK_i = \frac{VaR(X_i)}{\sum_i VaR(X_i)} \quad RAC(VaR)_i = AK_i \cdot RAC(Var)$$

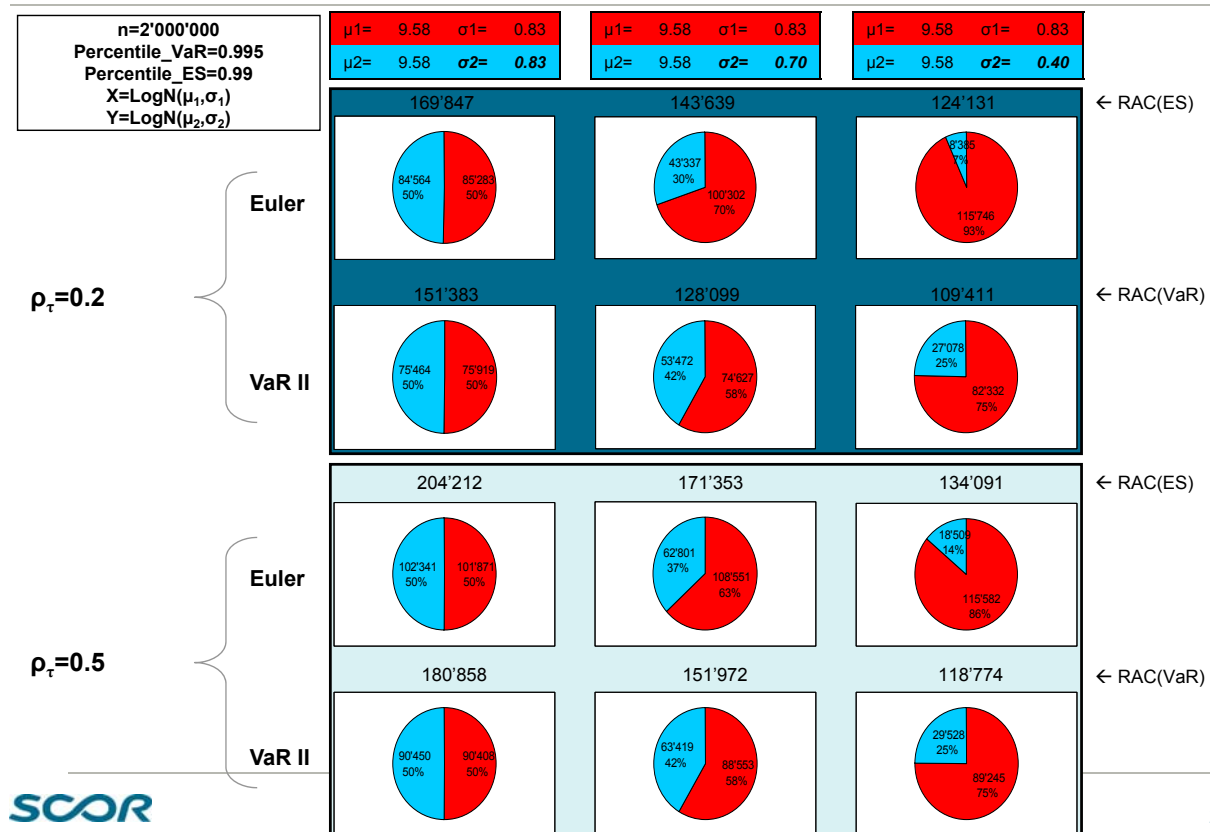
Numerical results

# simulation=2'000'000 Percentile_VaR=0.995 Percentile_ES=0.99 p _r =0.2 Copula=Clayton-M Z = X + Y X=LogN(μ ₁ , σ ₁) Y=LogN(μ ₂ , σ ₂)				
	μ ₁ =	9.58	9.58	9.58
	σ ₁ =	0.83	0.83	0.83
	μ ₂ =	9.58	9.58	9.58
	σ ₂ =	0.83	0.70	0.40
RAC (Value at Risk)		174'239	146'924	116'991
RAC (Expected Shortfall)		200'040	168'377	133'076
E U L E R	Percentage of RAC(ES) allocated to X	49.97%	63.88%	86.89%
	Percentage of RAC(ES) allocated to Y	50.03%	36.12%	13.11%
	RAC(ES) allocated to X	99'957	107'565	115'629
	RAC(ES) allocated to Y	100'083	60'812	17'447
V a R I	Percentage of RAC(VaR) allocated to X	57.00%	66.08%	80.12%
	Percentage of RAC(VaR) allocated to Y	57.00%	47.36%	26.47%
	RAC(VaR) allocated to X	99'325	97'087	93'733
	RAC(VaR) allocated to Y	99'320	69'584	30'967
V a R II	Percentage of RAC(VaR) allocated to X	50.00%	58.25%	75.17%
	Percentage of RAC(VaR) allocated to Y	50.00%	41.75%	24.83%
	RAC(VaR) allocated to X	87'122	85'584	87'939
	RAC(VaR) allocated to Y	87'117	61'340	29'052

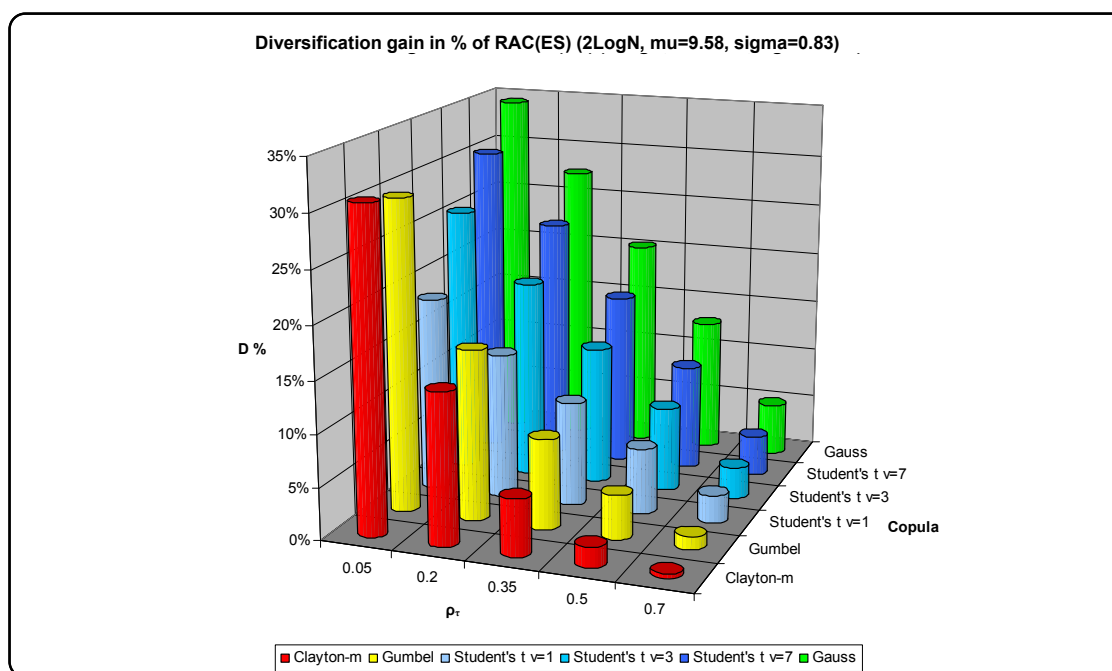
2 Lognormal - Copula: Clayton-m



2 Lognormal - Copula: Gaussian

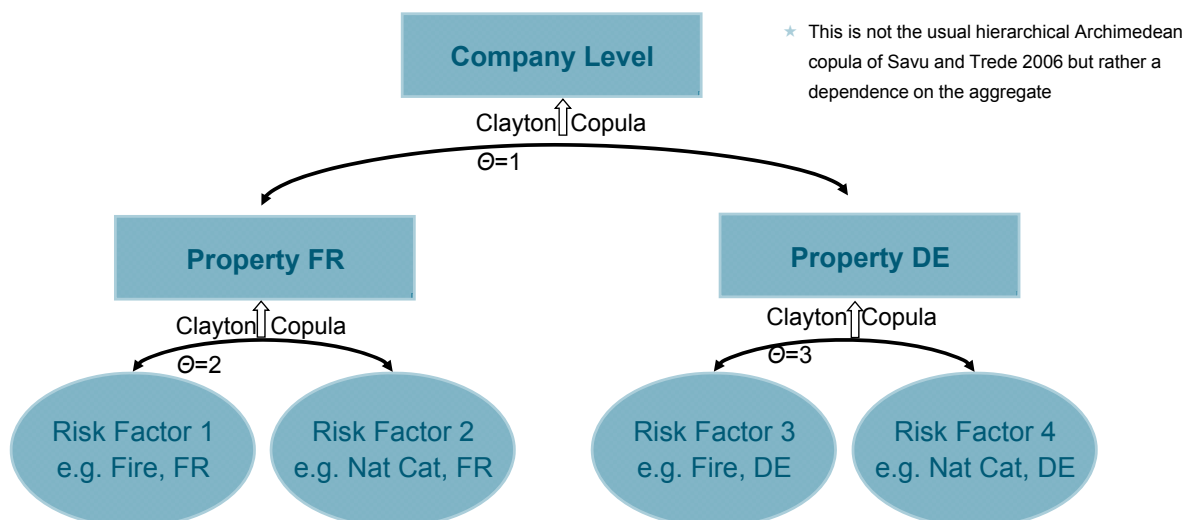


2 Lognormal - $D(ES, \rho_T, C)$



Hierarchical Dependence Tree

- **Base scenario:** Hierarchical Dependence Tree*
 - Hierarchy of 4 related marginal distributions
 - Using significantly different dependence parameters Θ



Scenarios: We fit the original dependence using the following copula scenarios and calculate the Diversification and RAC

- Hierarchical Dependence
 - Gumbel Hierarchy
 - Rank Correlation Hierarchy
 - Student-T Hierarchy
- Flat Dependence
 - Clayton Flat
 - Gumbel Flat
 - Rank Correlation Flat
 - Student-T Flat
- The base scenario is the *hierarchical Clayton* scenario presented before

Summary of the Statistical Results

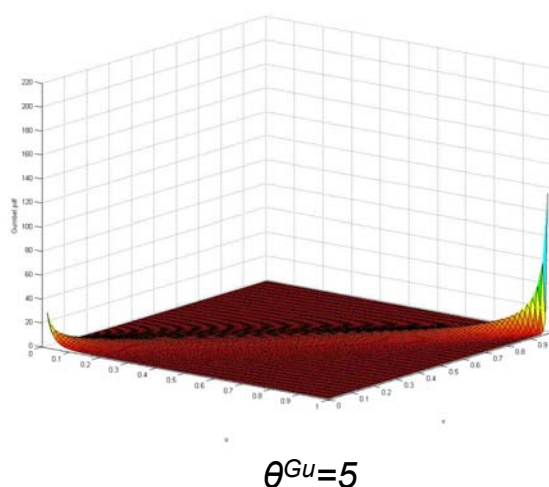
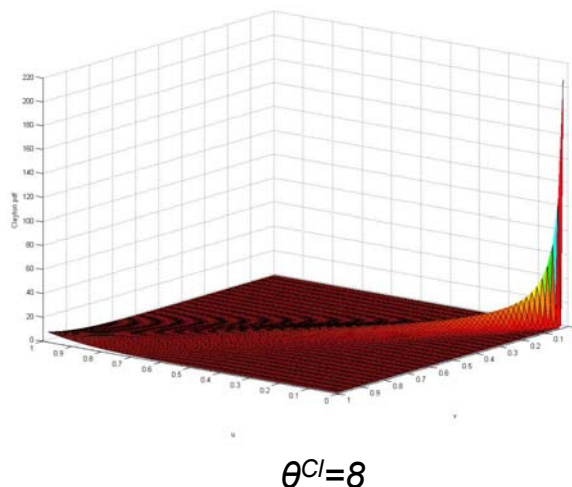
Statistical Results	250,000 simulations, Lognormal with $\mu = 10$ $\sigma = 1$							
	Clayton Hierarchy	Clayton Flat	Gumbel Hierarchy	Gumbel Flat	Rank Corr.	Rank Corr. Hierarchy	Student's T	Student's T Hierarchy
Expected	145'550	145'407	144'965	145'063	145'293	145'464	144'980	145'088
Std Dev	172'587	172'948	164'507	162'936	144'448	146'133	150'838	150'290
LH Div. Gain	9.56%	9.14%	5.70%	7.20%	3.98%	3.90%	3.20%	3.05%
RH VaR	1'070'462	1'078'190	1'021'144	1'028'533	870'389	877'052	901'757	909'335
RH Shortfall	1'248'146	1'251'138	1'195'423	1'188'803	977'617	990'103	1'043'301	1'045'099
RH RAC	1'102'595	1'105'731	1'050'458	1'043'740	832'324	844'639	898'322	900'012
RH Div. Gain	8.17%	7.73%	11.73%	12.69%	30.70%	30.33%	25.42%	25%

Dependence Model matters more than Dependence Structure

- We see that the elliptical copulas do not improve by moving from a flat structure to a hierarchical one.
- The Gumbel copula slightly improves if used in the appropriate dependence structure.
- The elliptical copulas grossly underestimate the risk and show undue diversification benefits.
- Gumbel copula is able to produce reasonable results on the left tail but also emphasizes a dependence on the right tail that does not exist in the benchmark model.

A comparison between Gumbel and Clayton-M

$$\rho_{\tau}=0.8$$



Fitting Convergence Plots: Methodology

The Fitting Convergence Plots are drawn using the following methodology:

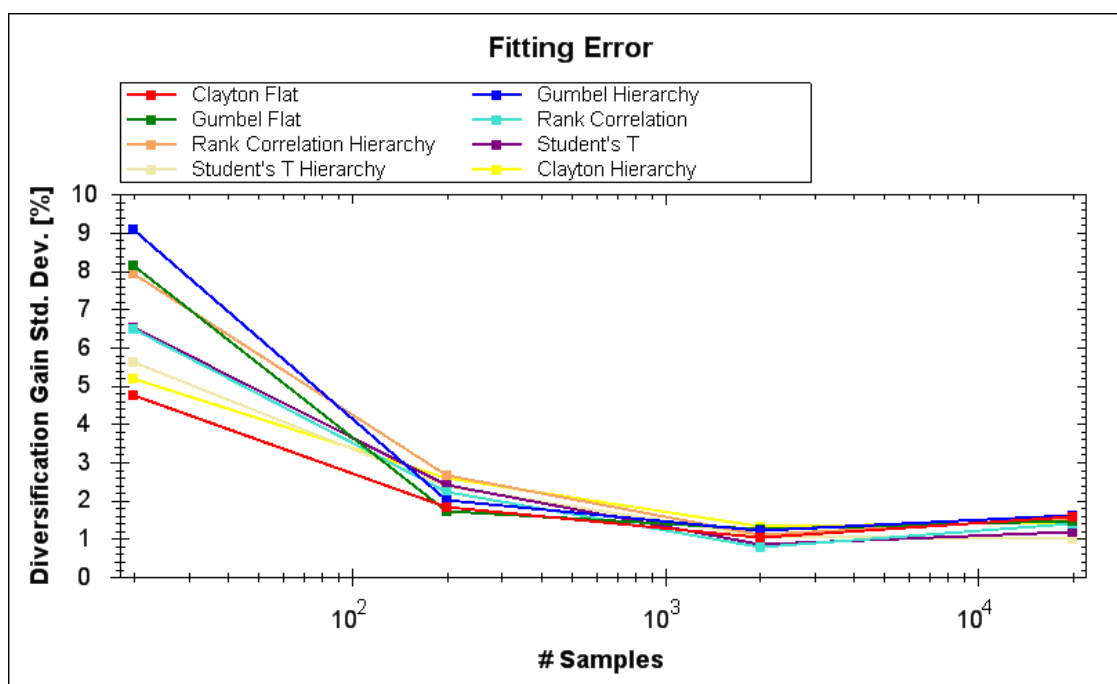
1. **Simulate** N observations from the reference scenario
2. **Fit** the corresponding scenario to the N observations
3. **Resample** the fitted scenario with 50'000 observations
4. **Measure** the diversification gain



The mean and standard deviation of the 10 runs per point are calculated.

- The fitting convergence plots show the mean $\pm$ one standard deviation for each N .
- The fitting error plots show the standard deviation for each N .

Standard Deviation of the Fits for 4 Marginals



Number of Observations matters less than Dependence Models

- We see that the elliptical copulas keep a systematic bias whatever the number of observations
- The Archimedean copulas fit much better the theoretical value with Clayton doing it the best, as expected
- The error of the estimation decreases with the number of observations and remains at a certain level even with 50'000 observations
- The structure of the dependence (hierarchical or flat) does not affect really the diversification benefit with hierarchical being slightly better for Archimedean copulas
- When the dependence is asymmetric (as it is usually the case for insurance liabilities), it is difficult to model it with symmetric dependence models (use asymmetric copulas)

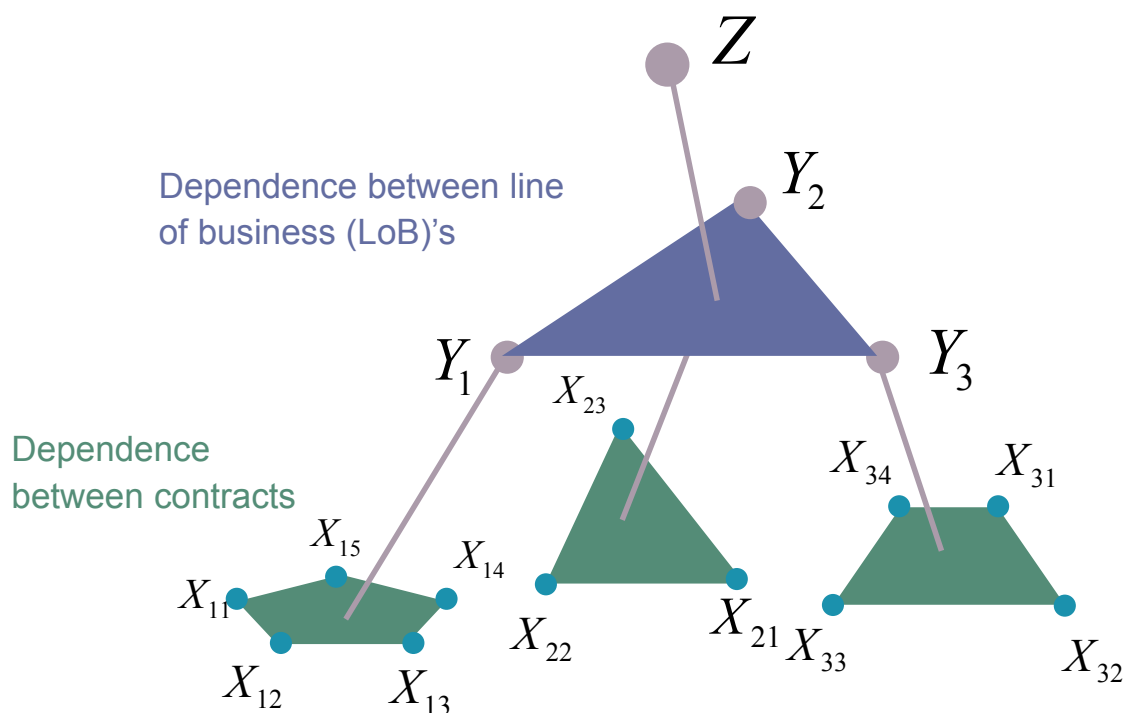
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Aggregating many risk factors

- Given a complex portfolio containing multiple risks, the development of a good concept of dependence requires *simplification assumptions*.
- To model all dependence between n risks, we would need to calibrate n^2 parameters. In other words, doubling the number of risks would mean quadrupling the number of dependences to evaluate.
- Overspecifying the dependence structure becomes a serious problem.
- We decide to use hierarchical dependence trees that reduce the number of parameters to a linear quantity related to the number of risks considered.

Hierarchical Dependencies



- Given this structure, the model is completely defined if we also require that:

$$P(X \leq x | Y = y, Z \leq z) = P(X \leq x | Y = y)$$

for all LoB $\mathcal{Y}$ and all risks $\mathcal{X}$ in $\mathcal{Y}$

- In other words, given that the result of $\mathcal{Y}$ influences the information about the result in $\mathcal{Z}$, the latter is not influenced by the distribution of $\mathcal{X}$ in $\mathcal{Y}$.

NORMA: Hierarchical Dependence

The screenshot displays the NORMA software interface with a hierarchical tree structure. The tree is organized into folders such as GAUM, Aviation, NonProportional, NewBusiness, Reserves, Credit + Surety, and Proportional. Each folder contains sub-nodes representing different risk components. A context menu is open over the tree, listing actions like 'Add Node', 'Delete Node', 'Copy Node', 'Paste Node', 'Expand Nodes', and 'Collapse Nodes'. Two callout boxes highlight the 'Administration of dependence model' and the ease of administering the model structure.

Name	Modelling Unit	Reinsuranc...	Cover ...	Business Maturity	Legal ...	Region
All	All	All	All	All	All	All
GAUM	GAUM	All	All	All	All	All
Aviation Non Gaum	Aviation	All	All	All	All	All
Treaty	Aviation	Treaty	All	All	All	All
NonProportional	Aviation	Treaty	Pro	All	All	All
NewBusiness	Aviation	Treaty	Pro	All	All	All
Scor CH incl. Asia and Aus branches	Aviation	Treaty	Pro	All	Scor C...	All
Reserves	Aviation	Treaty	Pro	All	All	All
Scor CH incl. Asia and Aus branches	Aviation	Treaty	Pro	All	Scor C...	All
Credit + Surety	Credit + Surety	All	All	All	All	All
Treaty	Credit + Surety	Treaty	All	All	All	All
Proportional	Credit + Surety	Treaty	Proporti...	All	All	All
NewBusiness	Credit + Surety	Treaty	Proporti...	NewBusiness	All	All
Scor CH incl. Asia and Aus branches	Credit + Surety	Treaty	Proporti...	NewBusiness	Scor C...	All
SCOR REASS. excl. UK, Italy, Germany and ...	Credit + Surety	Treaty	Proporti...	NewBusiness	SCOR ...	All
Reserves	Credit + Surety	Treaty	Proporti...	Reserves	All	All
Scor CH incl. Asia and Aus branches	Credit + Surety	Treaty	Proporti...	Reserves	Scor C...	All
SCOR REASS. excl. UK, Italy, Germany and ...	Credit + Surety	Treaty	Proporti...	Reserves	SCOR ...	All
NonProportional	Credit + Surety	Treaty	Proporti...	Reserves	SCOR ...	All
NewBusiness	Credit + Surety	Treaty	Proporti...	Reserves	SCOR ...	All
Scor CH incl. Asia and Aus branches	Credit + Surety	Treaty	Proporti...	Reserves	SCOR ...	All
SCOR REASS. excl. UK, Italy, Germany and ...	Credit + Surety	Treaty	Proporti...	Reserves	SCOR ...	All
Reserves	Credit + Surety	Treaty	Proporti...	Reserves	SCOR ...	All
Scor CH incl. Asia and Aus branches	Credit + Surety	Treaty	Proporti...	Reserves	SCOR ...	All
SCOR REASS. excl. UK, Italy, Germany and ...	Credit + Surety	Treaty	Proporti...	Reserves	SCOR ...	All

Copula between the Risk X and the Portfolio Z

- From the model for the LoB $\mathcal{Y}$ we get $C_{X,Y}$.
- From the distribution of the LoB $\mathcal{Y}$ and its copula to the portfolio Z , we get $C_{Y,Z}$.
- It is then possible, with relatively mild assumptions, to compute the copula between X and Z :

$$C_{X,Z}(u, w) = \int_0^1 \frac{\partial C_{X,Y}}{\partial v}(u, v) \frac{\partial C_{Y,Z}}{\partial v}(v, w) dv$$

Recall: Allocation of Capital to a Treaty

- C. Hummel (2002) showed that if we are given a treaty $\mathcal{S}$ of Z and the copula $C_{S,Z}$ between S and Z , then:

$$K_{\mathcal{S}} = -E\left[\mathcal{S} \mid Z \leq F_Z^{-1}(\alpha)\right] = -\int_{-\infty}^{+\infty} s d(H_S \circ F_S)(s)$$

with $H_S(u) = \frac{1}{\alpha} C_{S,Z}(u, \alpha)$

We call H_S the **Diversification Function** of $\mathcal{S}$ in Z .

- The distorted probability depends on the diversification of $\mathcal{S}$ within Z .
- Note that we do not need to know F_Z to calculate $K_{\mathcal{S}}$.

The Diversification Function

- Given the copula, $C_{X,Y}$, between the risk $\mathcal{X}$ and the LoB $\mathcal{Y}$, it is possible to define a **diversification function**, $H_X(u)$, as follows:

$$\begin{aligned}
 H_X(u) &= \alpha^{-1} C_{X,Z}(u, \alpha) \\
 &= \alpha^{-1} \int_0^1 \frac{\partial C_{X,Y}}{\partial v}(u, v) \frac{\partial C_{Y,Z}}{\partial v}(v, \alpha) dv \\
 &= \int_0^1 \frac{\partial C_{X,Y}}{\partial v}(u, v) dH_Y(v) \\
 &\approx \sum_{m=1}^M \frac{C_{X,Y}(u, v_{m+1}) - C_{X,Y}(u, v_m)}{v_{m+1} - v_m} (H_Y(v_{m+1}) - H_Y(v_m)).
 \end{aligned}$$

- Assuming that the grid $0 = v_0 < v_1 < \dots < v_M = 1$ is fine enough.

Only the Diversification Function *within* the LoB is Relevant

- We just saw that:

$$H_X(u) \approx \sum_{m=1}^M \frac{C_{X,Y}(u, v_{m+1}) - C_{X,Y}(u, v_m)}{v_{m+1} - v_m} (H_Y(v_{m+1}) - H_Y(v_m))$$

- From this expression, it follows:
 - ✓ To be able to price a risk within a line of business, we do not need to compute the copula between the different LoB's.
 - ✓ We only need to implement the diversification function, H_Y , with:

$$0 = v_0 < v_1 < \dots < v_M = 1$$

Diversification function: Direct approximation through Monte-Carlo simulation in practice

- ▶ In practice, we have an Internal Model performing a Monte-Carlo simulation of all risks including dependencies with, say, 100'000 simulations.
- ▶ Consider the shortfall sample of the total portfolio Z with a cumulative probability $F_Z(Z) < \alpha$ ($\alpha = 1\%$, normally, $\rightarrow$ shortfall sample of size 1'000)
- ▶ Each S value of a particular risk within the shortfall sample has a rank and thus an empirical cumulative probability v within the shortfall sample
- ▶ At the same time, each such S value has a rank in the total sample and thus an unconditional cumulative probability u ($= F_S(S)$)
- ▶ The diversification function H_s can be approximated by simply setting $v = H_s(u)$ for each S of the shortfall sample

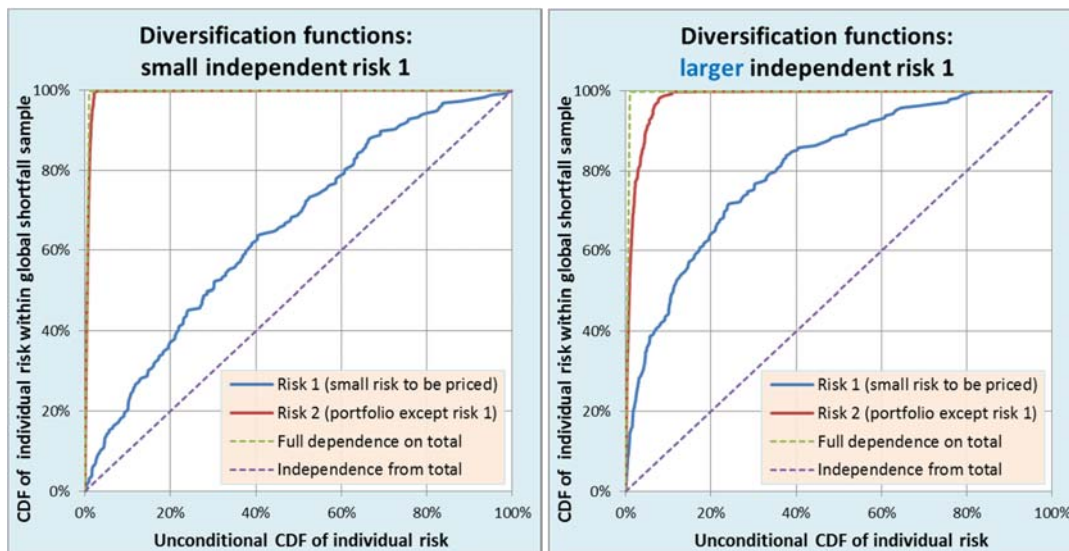
Diversification function: computed examples, varying the size of risks

Excel examples with just 20'000 simulations. Here assuming normally distributed risks

Exploring the influence of the **size** of a risk to be priced, relative to the total portfolio

$\rightarrow$ Small independent risks have lower diversification functions, close to the diagonal line

We need a huge number of simulations to obtain smooth, accurate diversification functions, especially for small risks (maybe, millions!).

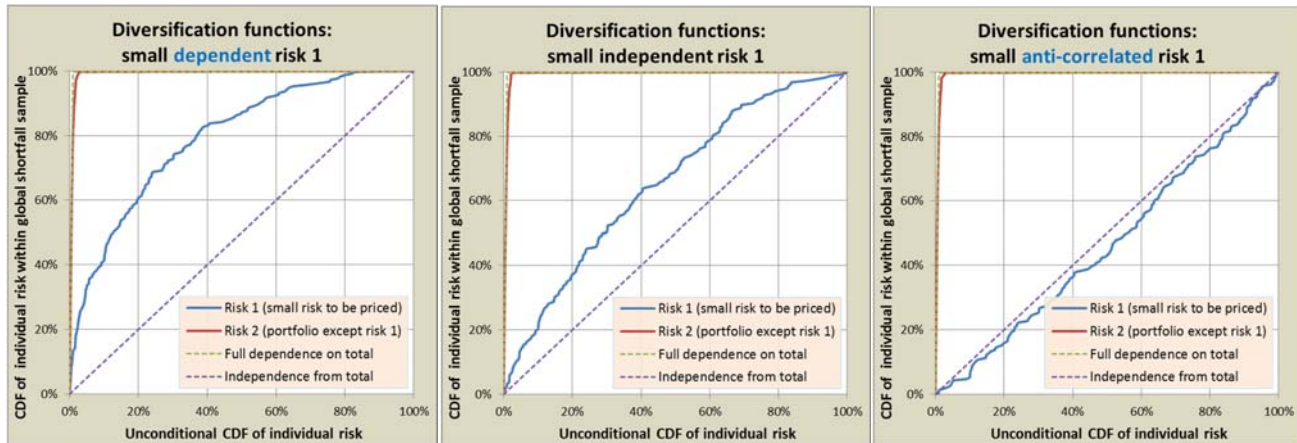


Diversification function: computed examples, varying the dependence between risks

Excel examples with just 20'000 simulations. Here assuming normally distributed risks

Exploring the influence of the **dependency** (here: linear correlations 25%, 0%, -25%) between risks on the form of diversification functions

→ Dependent risks have a higher diversification function; for anti-correlated risks the function may lie below the diagonal line



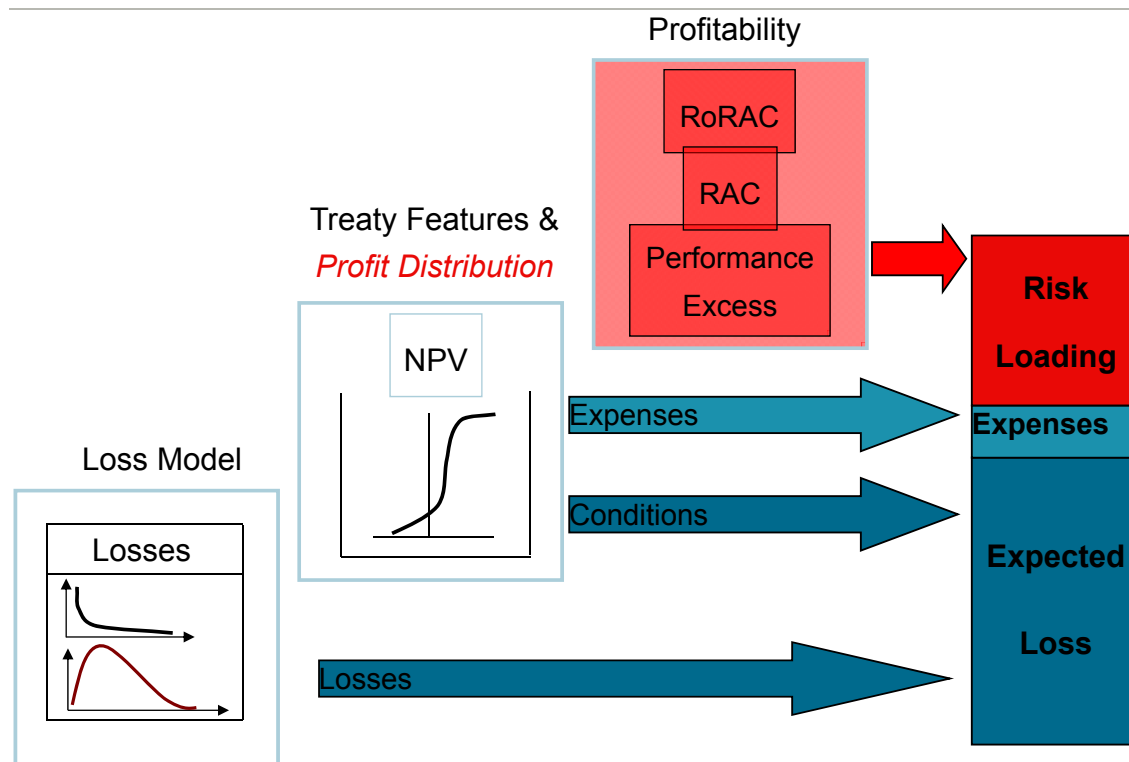
Main application of the diversification function: computing the diversified RAC

- ▶ The diversification function can be used to calculate the shortfall contribution K_s (→ diversified RAC) of a risk S directly. RAC is also needed for pricing (cost of capital)
- ▶ **Method 1:** (Numerical) integration over probability v within shortfall sample
→ $K_s = - \int_0^1 F_s^{-1}[H_s^{-1}(v)] dv$
Requires inverse distribution F_s^{-1} and inverse diversification function H_s^{-1} in numerical approximation
- ▶ **Method 2:** (Numerical) integration over unconditional probability u
→ $K_s = - \int_0^1 F_s^{-1}(u) [dH_s(u) / du] du$
Requires the inverse distribution F_s^{-1} and the (numerical) derivative of the diversification function $H_s(u)$
- ▶ **Method 3:** (Numerical) integration over the value s of the risk S (as in the original definition)
→ $K_s = - \int_{-\infty}^{\infty} s \{dH_s[F_s(s)] / ds\} ds$
Numerically demanding due to infinite integration limits and the complex derivative in the formula

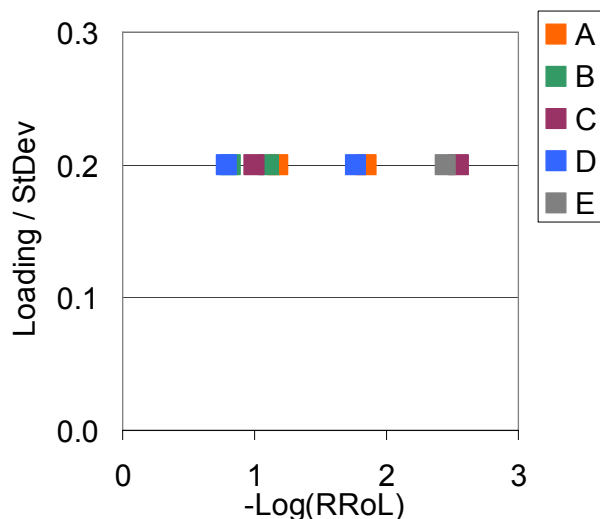
Main advantage of the diversification function: direct “local” RAC computation

- ▶ Main **advantage**: Once the **diversification function** for a (small) risk category has been computed, we can compute the Risk-Adjusted Capital (RAC) of that risk immediately **without reconsidering the whole portfolio**
- ▶ This is even true, in good approximation, if the distribution of the (small) risk is **new**, i.e. **different** from the one used in the **original diversification function calculation** (but assuming an unchanged dependency of the new risk from other risks)
- ▶ These advantages make the diversification function a useful tool in (P&C) **pricing**:
 - New treaties to be priced usually constitute **small** risks as compared to the required capital for the total portfolio
 - New treaties to be priced usually have **new** ultimate value **distributions** (not exactly identical to those used when determining the original diversification function)
 - New treaties have to be priced over the whole business year – also at times when no-one has time for a re-evaluation of the total portfolio

Coming up with a Quotation



Risk Loading for Various CAT Programs

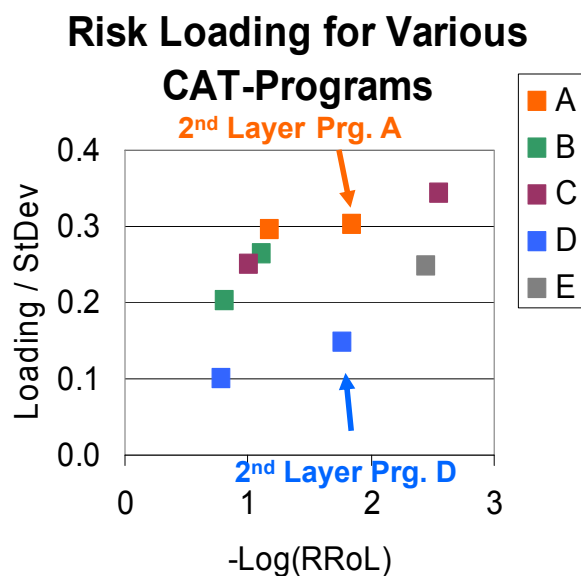


- Using the standard deviation loading makes all these programs lie on a straight line since they present very similar risk characteristics.

- Risk Rate on Line

$$RRoL = \frac{\text{Expected Loss}}{\text{Granted Limit}}$$

Active Portfolio Management: An Example

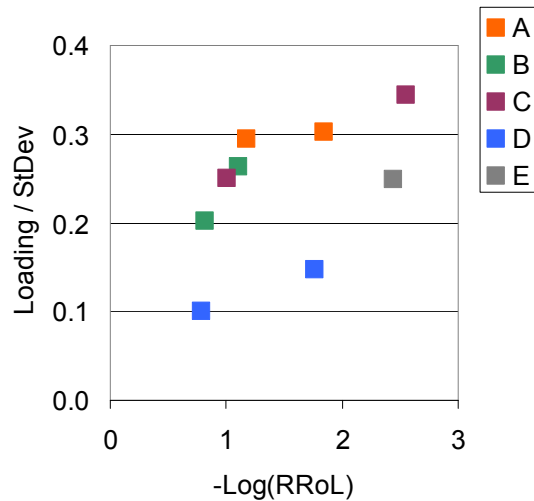


- The capital allocation taking into account the *diversification effects* within the portfolio results in *different loading* for similar risks.

- Example:

- The distribution of the second layer **A** and **D** are almost identical.
- **A** presents a *stronger*, **D** a *weaker* dependence to the rest of the portfolio.

Risk Loading for Various CAT-Programs



- Diversification or risk accumulation are favoured respectively penalized in the price.
- As a result, *the pricing mechanism implicitly optimizes the portfolio.*

Agenda

- 1 Copula a general model for dependence
- 2 Various forms of copula and their influence on diversification
- 3 Capital allocation and dependence
- 4 Hierarchical tree to model a large number of risks
- 5 Conclusions

Conclusion

- The RAC computation depends not only on the choice of a risk measure and on a model for the risks but also on the **dependence model** for the interaction between those risks.
- Realistic dependence model for insurance and reinsurance must be non-linear: dependence increases in times of stress.
- A good way to model non-linear dependence is to use copulas.
- Insurance makes only sense through diversification of the risks. Thus it is important to **model the entire portfolio** to achieve a reasonable price for the risks.
- A sound capital allocation methodology allows to price the risk of an insurance contract to provide the appropriate Return on Equity.

References

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*) on demand at <http://www.scor.com/en/sgrc/scor-publications/scor-papers.html>

Appendix

An alternative correlation measure: Kendall's tau

- For the random pair (X, Y) the Kendall's tau is defined as:

$$\rho_{\tau} = \mathbf{P}\{(X - X')(Y - Y') > 0\} - \mathbf{P}\{(X - X')(Y - Y') < 0\},$$

where (X', Y') is an independent copy of (X, Y) .

- It is the probability of concordance minus the probability of discordance.
- It can be shown to be a copula-only based quantity and has a unique expression for various forms of copula.
- It is thus very useful to express a benchmark dependence to compare various copulas.

Some formulas we used in our analysis:

Kendall's tau for elliptical copulas $\rightarrow \rho_\tau = \frac{2}{\pi} \arcsin \rho$

Kendall's tau for Gumbel copula $\rightarrow \rho_\tau = 1 - \frac{1}{g}$ hence $g = \frac{1}{1 - \rho_\tau}$

Kendall's tau for Clayton copula $\rightarrow \rho_\tau = \frac{g}{g+2}$ hence $g = \frac{2\rho_\tau}{1 - \rho_\tau}$