

Parametric Mortality Indexes: From Index Construction to Hedging Strategies

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Introduction
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Construction of Mortality Indexes
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Hedging Strategies
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Conclusion
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Introduction

Construction of Mortality Indexes

Securitization

Hedging Strategies

Conclusion

Life Market

- ▶ the market for mortality/longevity-linked securities
- ▶ trading/hedging of longevity risk
- ▶ Basel Committee's Joint Forum (2013):
 - ▶ the PV of longevity-linked liabilities would increase by 3-4% if the life expectancy of a DB plan member increases by 1 year
- ▶ CRO Forum (2010):
 - ▶ the global longevity risk exposure: \$15-\$25 trillion
- ▶ underestimation of 1 year life expectancy implies extra liabilities of \$450 billion to \$1 trillion

Longevity Risk Transfer

- ▶ Swiss Re's mortality catastrophe bond (2003)
- ▶ EIB/BNP/PartnerRe longevity bond (2004)
- ▶ q-forward between J.P. Morgan and Lucida (January 2008)
- ▶ longevity swap between Canada Life and hedge/ILS funds with J.P. Morgan as the intermediary (July 2008)
- ▶ 29 longevity swaps in the UK up until 2014: £55bn
- ▶ the largest deal so far: General Motors transferred \$26bn pension liabilities to Prudential (November 2012)

Challenges (and Opportunities)

- ▶ challenge: lack of liquidity
- ▶ 1 Feb 2010: Life and Longevity Markets Association (LLMA)
 - ▶ Aviva, AXA, Deutsche Bank, J.P. Morgan, Morgan Stanley, Prudential PLC and Swiss Re.
 - ▶ aim: to promote the development of a liquid trading market for longevity risks.
- ▶ solution: creation of mortality/longevity indexes

Mortality Indexes

- ▶ model-free indexes
 - ▶ Credit Suisse Longevity Index (Credit Suisse, 2005), LifeMetrics Index (J.P. Morgan, 2007), Q×X index (Goldman Sachs, 2007)
 - ▶ these indexes are either highly aggregate or specific
 - ▶ keep track of a large number of indexes
- ▶ model-based indexes
 - ▶ stochastic models
 - ▶ time-varying parameters

Model-based Mortality Indexes

- ▶ 3 primary criteria by Chan et al. (2014)
 - ▶ the new-data-invariant property: to ensure tractability
 - ▶ highly interpretable
 - ▶ varying age-pattern of mortality improvement
- ▶ six stochastic models in Dowd et al. (2010)
 - ▶ M1 (Lee-Carter model)
 - ▶ M2 (Renshaw-Haberman model)
 - ▶ M3 (Age-Period-Cohort model)
 - ▶ M5 (Cairns-Blake-Dowd model)
 - ▶ M6 (Cairns-Blake-Dowd model with cohort effects)
 - ▶ M7 (Cairns-Blake-Dowd model with cohort and quadratic age effects)
- ▶ Chan et al. (2014) found that M5 is the most suitable model

Mortality Models

- ▶ M1: Lee and Carter, 1992

$$\ln(m_{x,t}) = \beta_x^{(1)} + \beta_x^{(2)} \kappa_t^{(2)}$$

- ▶ M2: Renshaw and Haberman, 2006

$$\ln(m_{x,t}) = \beta_x^{(1)} + \beta_x^{(2)} \kappa_t^{(2)} + \beta_x^{(3)} \gamma_{t-x}^{(3)}$$

- ▶ M3: Osmond, 1985

$$\ln(m_{x,t}) = \beta_x^{(1)} + n_a^{-1} \kappa_t^{(2)} + n_a^{-1} \gamma_{t-x}^{(3)}$$

- ▶ M5: Cairns et al., 2006

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x})$$

- ▶ M6: Cairns et al., 2009

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) + \gamma_{t-x}^{(3)}$$

- ▶ M7: Cairns et al., 2009

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) + \kappa_t^{(3)}((x - \bar{x})^2 - \hat{\sigma}_x^2) + \gamma_{t-x}^{(4)}$$

Adapting Mortality Models

- ▶ model with age-specific parameters (M1)
 - ▶ estimate the model parameters using a restricted sample period $[t_{\text{start}}, t_{\text{mid}}]$
 - ▶ keep the age-specific parameters fixed when we update the model for sample period $[t_{\text{mid}}, t_{\text{end}}]$
- ▶ model with cohort effect parameters (M6 and M7)
 - ▶ estimate the time-varying parameters first
 - ▶ estimate the cohort effect parameters from the residuals

Adapting Mortality Models

- ▶ model with both age-specific and cohort effect parameters (M2 and M3)
 - ▶ keep the age-specific parameters fixed
 - ▶ estimate the time-varying parameters first
 - ▶ estimate the cohort effect parameters from the residuals

Adapting Mortality Models

- ▶ M1*: adapted M1

$$\ln(m_{x,t}) = \beta_x^{(1)*} + \beta_x^{(2)*} \kappa_t^{(2)}$$

- ▶ M2*: adapted M2

$$\ln(m_{x,t}) = \beta_x^{(1)*} + \beta_x^{(2)*} \kappa_t^{(2)} + \beta_x^{(3)*} \gamma_{t-x}^{(3)*}$$

- ▶ M3*: adapted M3

$$\ln(m_{x,t}) = \beta_x^{(1)*} + n_a^{-1} \kappa_t^{(2)} + n_a^{-1} \gamma_{t-x}^{(3)*}$$

- ▶ M6*: adapted M6

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) + \gamma_{t-x}^{(3)*}$$

- ▶ M7*: adapted M7

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) + \kappa_t^{(3)}((x - \bar{x})^2 - \hat{\sigma}_x^2) + \gamma_{t-x}^{(4)*}$$

Constructing Mortality Indexes

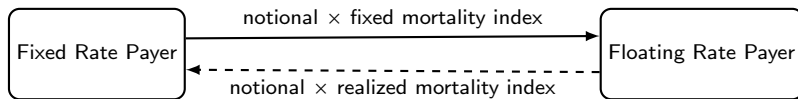
- ▶ gender-specific and unisex mortality data from 8 populations
 - ▶ Australasia: Australia
 - ▶ East Asia: Japan
 - ▶ Nordic region: Norway, Sweden
 - ▶ Western Europe: England and Wales, France
 - ▶ North America: Canada, United States
- ▶ data source: Human Mortality Database
- ▶ age range: 40-90
- ▶ data sample period: [1950,1979], [1980,2009]

Constructing Mortality Indexes

- ▶ maximum likelihood estimation (MLE)
- ▶ model selection criterion
 - ▶ reduction in log-likelihoods between original model and adapted model
 - ▶ Bayesian Information Criterion (BIC)
- ▶ M7* gives the best BIC values and the smallest reductions in log-likelihood values
- ▶ M7* performs the best in a backtest that compares the projected cohort death probabilities with the actual values
- ▶ construct mortality indexes using $\kappa_t^{(1)}, \kappa_t^{(2)}, \kappa_t^{(3)}$ in M7*
- ▶ highly interpretable: intercept, slope, curvature parameter
- ▶ varying age-pattern of mortality improvement

K-forward

- ▶ standardized mortality-linked security
- ▶ a swap between a fixed amount (pre-determined forward value) and a random amount (realized index value) related to one of the three indexes in a reference year
- ▶ K1-forward, K2-forward, K3-forward



$$Y \times (\tilde{\kappa}_{t^*}^{(i)} - \kappa_{t^*}^{(i)}), \quad i = 1, 2, 3.$$

Key K-duration (KKD)

- ▶ similar to key q-duration (Li and Luo, 2012) for q-forwards
- ▶ 'key': K-forwards are only available in certain key years
 $t_1, t_2, \dots, t_n$
- ▶ measures the change in the value of a liability with respect to a small change in a key K-index
- ▶ two assumptions
 - ▶ a shock in $\kappa_{t_j}^{(i)}$ is accompanied by a level shift in $\kappa_t^{(i)}$ over the period of $t_j \leq t < t_{j+1}$
 - ▶ the shock on $\kappa_t^{(i)}$ has no impact on $\kappa_t^{(h)}$ for all $i \neq h$ and t
- ▶ $KKD_i(P(\kappa), j) = \frac{\partial P(\kappa)}{\partial \kappa_{t_j}^{(i)}}$

Building Longevity Hedge

- ▶ KKD strategy
- ▶ KKD of liability portfolio is estimated numerically
- ▶ KKD of K-forward can be derived analytically
- ▶ KKD of liability portfolio = KKD of hedge portfolio consisting of K-forwards, for each key K-index in each key year
- ▶ determine the required notional amounts of K1-forward, K2-forward and K3-forward separately in respective key years

Hedging Illustrations: Single Cohort

- ▶ pension plan coverage: \$1 at the beginning of each year from age 65 until the pensioner dies or attains age 91
- ▶ mortality data: English and Welsh males, ages 40-90, [1950,2009]
- ▶ reference years: 2015, 2020, 2025, 2030
- ▶ interest rate: 3% flat
- ▶ parametric bootstrap (see Brouhns et al., 2005) simulation: 5000 scenarios
- ▶ amount of longevity risk reduction:

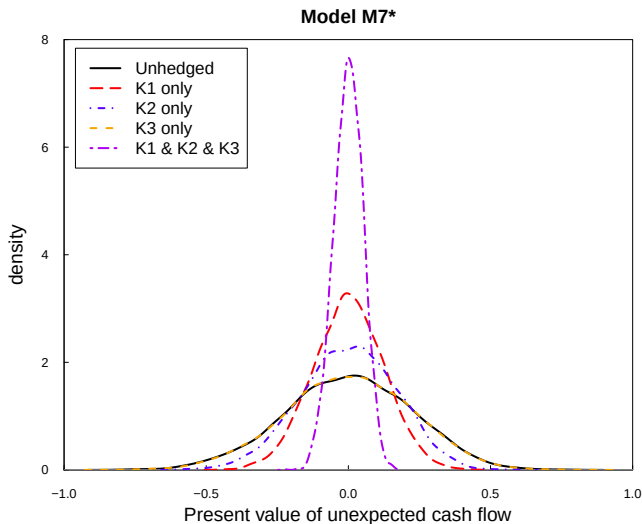
$$R = 1 - \frac{\text{variance of PV of unexpected cash flows after hedging}}{\text{variance of PV of unexpected cash flows without hedging}}$$
- ▶ simulation models: M7*, M5, M3, M2, MRW (Bell, 1997)

Hedging Illustrations: Single Cohort

- ▶ KKD strategy: simple calibration
- ▶ optimal hedge: simulations + numerical optimization

Simulation model	KKD strategy	Optimal hedge
M7*	94.7%	97.3%
M5	96.0%	99.1%
M3	95.6%	96.6%
M2	95.2%	95.8%
MRW	93.5%	94.0%

Hedging Illustrations: Single Cohort



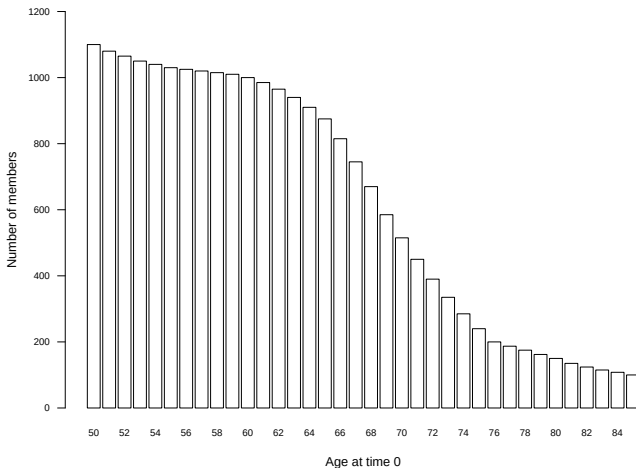
Hedging Illustrations: Single Cohort

- ▶ Sampling risk (small-sample risk): smaller R for smaller number of pensioners
- ▶ Sensitivity tests
 - ▶ interest rate: R is not sensitive to the interest rate assumption
 - ▶ availability of K-forwards: more key years and/or smaller separation between two adjacent key years produce more effective hedge
 - ▶ age range: R of K1- and K3-forward increase for an older age range, but R of K2-forward drops
- ▶ advanced ages: satisfactory R for pension coverage until age 101

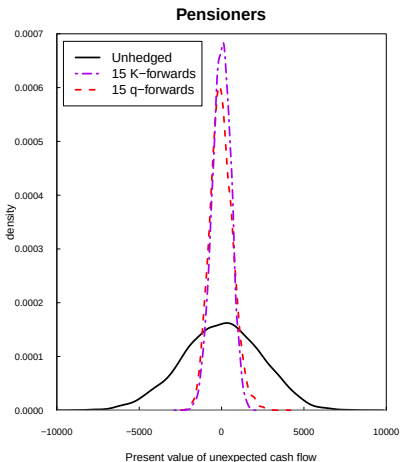
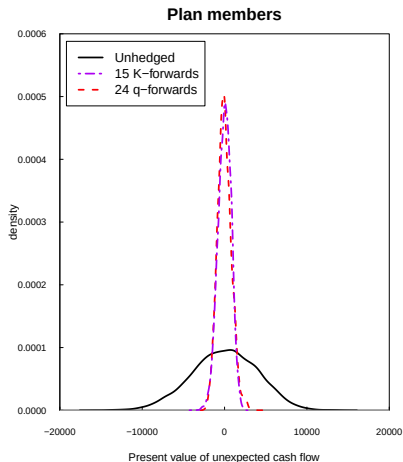
Hedging Illustrations: Multiple Cohorts

- ▶ consider a multi-cohort pension plan with a coverage from age 60 to 91
- ▶ with both active members (ages 50-59) and retirement pensioners (ages 60-90)
- ▶ compare K-forward hedge with q-forward hedge
- ▶ K-forward: reference year
- ▶ q-forward: reference age and reference year
- ▶ K-forward hedge is easier to calibrate using key K-index
- ▶ q-forward hedge requires key cohorts and key q-rates

Hedging Illustrations: Multiple Cohort



Hedging Illustrations: Multiple Cohort



Hedging Illustrations: Multiple Cohorts

- ▶ the number of instruments required to produce a satisfactory hedge using K-forward remains the same, but that of q-forward rises with a larger number of cohorts
- ▶ due to the reference rates of K-forward and q-forward contracts
- ▶ K-forwards are potentially more liquid than q-forwards

Concluding Remarks

- ▶ adapting mortality models to achieve the new-data-invariant property
- ▶ constructing mortality indexes using model M7*
- ▶ securitization of K-forward
- ▶ KKD hedging strategy yields excellent longevity risk reduction
- ▶ K-forwards are potentially more liquid for hedging purpose

Future Research

- ▶ convexity measure in calibrating the hedge
- ▶ dynamic hedging
- ▶ population basis risk

Q&A

Thank you!